CS3911 Introduction to Numerical Methods with Fortran Exam 2 Fall 2007

100 points – 6 pages

Name: ____________________________

- *Justify your answer with a convincing argument*. If there is no justification when it is needed, you will receive ZERO point for that question even though you have provided a correct answer. *I consider a good and correct justification more important than a right answer. Thus, if you provide a very vague answer without a convincing argument to show your answer being correct, you will likely receive a very low grade.*
- Do those problems you know how to do first. Otherwise, you may not be able to complete this exam on time. If you follow our classroom discussions and understand the most basic components, you should be able to quickly complete more than 60% of this exam. The remaining problems, however, test if you are able to apply and use the basics properly.
- To avoid confusion in grading, all answers must have at least five significant digits.

1. **Systems of Linear Equations**

(a) **[6 points]** Answer the following **two** questions: **(1)** define the meaning of a *diagonal dominant* matrix $A = [a_{i,j}]_{n \times n}$; and **(2)** what is the advantage(s) for a matrix to be diagonal dominant?

(b) **[20 points]** Let ϵ be a very small positive number (*i.e.*, $\epsilon \approx 0$). Do the following **two** problems: **(1)** solve the following system of linear equations *with* **and** *without* partial pivoting; and **(2)** find the major reason or reasons that can explain the difference(s) between the two solutions. **You have to clearly state your findings with a convincing argument. Just stating a "reason" such as "it is because of cancelation" or "overflow" will receive zero point.**

$$\begin{bmatrix} \epsilon & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(c) **[15 points]** Given matrix A as shown below, find its LU-decomposition *without* pivoting. **You have to show all computation steps, and explain how you get the results. Otherwise (*e.g.*, providing an answer only and/or asking me to guess your intention from a bunch of numbers), you will receive zero point.**

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 2 & 4 & 6 \end{bmatrix}$$

(d) **[12 points]** Use Gauss-Seidel method to solve the following system of linear equations, and fill the table below with your results. The initial value (*i.e.*, iteration 0) is $x = y = z = 0$, and you only do two iterations (*i.e.*, iterations 1 and 2).

$$\begin{array}{rcrcrcl} 4x & + & y & + & z & = & 1 \\ x & + & 4y & + & z & = & 2 \\ x & + & y & +4 & z & = & 4 \end{array}$$

Iteration	x	y	z
0	0	0	0
1			
2			

(e) **[20 points]** Suppose a program read in the following system of linear equations:

$$A \cdot x = B \qquad \text{where } A = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

and delivered the solution $x = y = 1$. However, this solution is obviously inaccurate. Suppose we happen to know the LU-decomposition of A as shown below. Use the iterative refinement method to improve the accuracy of this "solution." **You have to show <u>all</u> computation steps, and explain how you get the results. Otherwise (*e.g.*, only providing an answer and/or asking me to guess your intention from a bunch of numbers), you will receive <u>zero</u> point.**

$$A = L \cdot U = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$$

2. **Eigenvalues and Eigenvectors**

(a) **[12 points]** Use the power method to find the largest eigenvalue and its corresponding eigenvector of matrix A as shown below, and fill the following table with your results. The initial value (*i.e.*, iteration 0) is $z = [1, 1]^T$, and you only do two iterations (*i.e.*, iterations 2 and 3).

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

Iteration	*Approx.* *Eigenvalue*	*Approx. Eigenvector* x	y
0	1	1	1
1			
2			

(b) **[15 points]** Use Jacobi method to find *all* eigenvalues and their corresponding eigenvectors of the following symmetric matrix A. **You should provide clearly all computation details, and match each eigenvalue with its corresponding eigenvector. Otherwise, you will risk low grade. Additionally, you will receive zero point if you do not use Jacobi method.**

$$A = \begin{bmatrix} 2 & -2\sqrt{3} \\ -2\sqrt{3} & 6 \end{bmatrix}$$

Grade Report

Problem		*Possible*	*You Received*
1	a	6	
	b	20	
	c	15	
	d	12	
	e	20	
	a	12	
	b	15	
Total		100	