Computing with Geometry as an Undergraduate Course

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Why Is It Important?

- This is a *geometric* world

- *Geometry* plays a central role in many areas in computer science and engineering:
  
  computer graphics, computer-aided design, geometric modeling, computer vision, manufacturing, robotics, GIS, ...

- The skill of handling geometric objects is virtually missing in a typical computer science curriculum

- The Computer Science and Telecommunication Board and National Research Council suggested that more *continuous* math should be taught in CS
Design Merit

- The underlying *theme* is Geometry Representation Algebra Algorithm Program

- Intuitive, less mathematical and elementary

- Hands-on and learning-by-doing

- Using **DesignMentor**: A tool for teaching curve and surface design

- Prerequisites: calculus and linear algebra
Unit 1: Course Overview

- The theme of this course
- The complexity of a geometric problem
  - Dimensional, analytic, combinatorial
- The impact of float point calculation on geometric problems

A poor equation solver can achieve this easily!!
Unit 2: Basic Geometric Concept

- **Euclidean Geometry**
  - Coordinate, Euclidean transformations and their matrices, quaternions

- **Affine Geometry**
  - Affine transformations (e.g., scaling, shear) and their matrices, affine space

- **Projective Geometry**
  - The concept of points and lines at infinity, projective transformations and their matrices, projective space, cross−ratio

- **Floating Point Computation**
  - Lossing of significant digits, error cumulation, problems with commutative law and distributive law

- **Examples and Applications**
Unit 3: Object Representations

Wireframes:
Advantages, disadvantages, ambiguity

Boundary Representations:
Manifolds, winged-edge representations, Euler–Poincare characteristic, Euler operators, non-manifolds

Constructive Solid Geometry:
Interior, exterior and closure, regularized Boolean operators, CSG solid design

Student work on CSG Design
Unit 4: Parametric Curves and Surfaces

- Polynomial and rational curves
- The moving triad
  - tangent, bi-normal, normal vectors
- Curvature and curvature sphere
- Singular and Inflection Points
- Tangential and Geometric Continuity
- The Meaning of Uniformization
  - Not all curves are polynomial

Discussions
  - Classification of conics using the line at infinity
  - The proof of circle being non-polynomial
Unit 5: Bezier, B–spline and NURBS Curves

- **Motivation and Important Properties:**
  - Control points, partition of unity, convex hull, variation diminishing and affine invariance

- **Bezier Curves:**
  - Construction, editing, de Casteljau’s algorithm, derivatives, subdivision and degree elevation

- **B–spline Curves**
  - B–spline basis, construction, local modification, strong convex hull, derivatives

- **NURBS** *Non–Uniform Rational B–Spline*
  - Motivation, meaning of weights, NURBS basis, a NURBS curve as the projection of a 4D B–spline curve to 3D, infinite control points
Unit 6: Advanced Geometric Algorithms

- Knots Revisited
  The meaning of knots

- Knot Insertion
  Inserting a new knot *without* changing the shape of the curve, single insertion, multiple insertion

- De Boor’s Algorithm
  De Boor’s algorithm via knot insertion, and de Casteljau’s algorithm as a special case

- Curve Subdivision

- Degree Elevation
Unit 7: Parametric, Bezier, B–Spline, and NURBS Surfaces

Basic Concepts
Surface normal, tangent plane, naive surface triangulation, isoparametric curves, tensor product surfaces

Bezier, B–Spline and NURBS Surfaces
Surface construction from two curves, 2D basis functions, 3D important properties from those of 2D, de Casteljau’s and de Boor’s algorithms for surfaces

A Twisted Sphere

Dini’s Surface

Naive Surface Triangulation: Student Work
Unit 8: Cross–Sectional Design

What is Cross–Sectional Design
Creating surfaces using curves, profile and trajectory curves, compatible curves

Cross–Sectional Design Surfaces
Ruled surfaces, surfaces of revolution, swung surfaces, simple swept surfaces, and skinned surfaces

Interpolation Surfaces
Swept surfaces via skinning, interpolating a curve network (i.e., Gordan surfaces)

Generating a surface of revolution
Unit 9: Algorithm Robustness

- Loss of Significant Digits
  Imprecise input, cumulation of errors in geometric transformations and computation

- Various Computation Schemes
  Exact (Symbolic), Approximation, and Interval arithmetic

- Robust Algorithm Design

- Well-Known Experiments
  Equation solvers, Sturm sequences, Dobkin’s growing/shrinking pentagons, Euclidean transformations
Course Evaluation

This elective course has been taught three times to junior/senior students

The following is a student self assessment survey

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17 questions were asked before and after the course

Self assessment levels range from 0 (no understanding) to 4 (excellent understanding)

The average gain is statistically significant

Students like the non–mathematical approach and our software tools
Results and Dissemination

Available Materials

A course electronic book, a software tool DesignMentor, a set of user guides and publications


Dissemination Statistics

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There are 900+ downloads of DesignMentor

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Future Work

- Interpolation and Approximation
  Regular and scattered data

- Triangular Patches
  Bezier triangles, triangular B–splines, and multi–sided patches

- Curve and Surface Interrogation

- Implicit Curves and Surfaces

- The Blossoming Principle

- Important Geometric Operations
  Surfaces blending, curve and surface intersection, curve tracing, ...