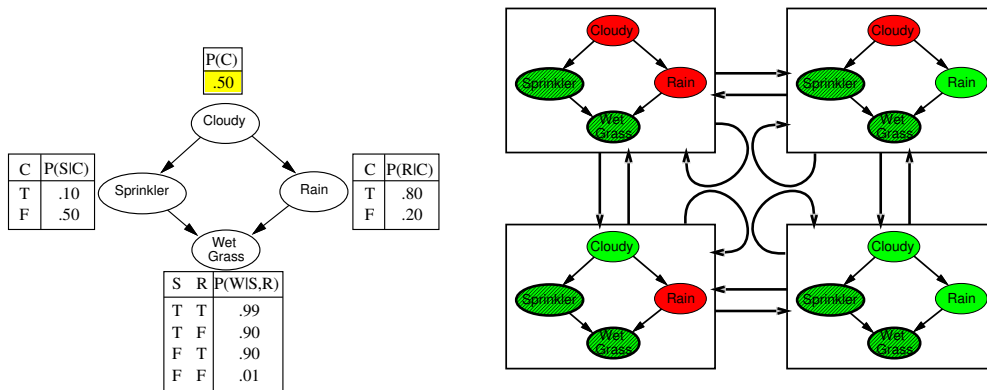


CS5811 handout

The procedure to obtain the sampling distributions for MCMC



The probability of a variable given its Markov blanket is proportional to the probability of the variable given its parents times the probability of each child given its respective parents:

$$P(x'_i | mb(X_i)) = \alpha P(x'_i | \text{parents}(X_i)) \times \prod_{Y_j \in \text{children}(X_i)} P(y_j | \text{parents}(Y_j))$$

Consider the query $\mathbb{P}(R | S = t, W = t)$.

S is true from the evidence. Suppose that R is true in the state. We will be sampling for C .

The Markov blanket of C is its parents (\emptyset), its children ($\{R, S\}$), and the other parents of its children (\emptyset). We use the following distributions to sample C .

$$\begin{aligned} \mathbb{P}(C | MB(C)) &= \mathbb{P}(C | R = t, S = t) = \alpha \mathbb{P}(C) \mathbb{P}(S = t | C) \mathbb{P}(R = t | C) \\ &= \alpha \langle 0.5, 0.5 \rangle \langle 0.1, 0.5 \rangle \langle 0.8, 0.2 \rangle \\ &= \alpha \langle 0.04, 0.05 \rangle \\ &= \langle \frac{4}{9}, \frac{5}{9} \rangle \end{aligned}$$

For the states where R is false, $\mathbf{P}(C | \neg R, S)$ is calculated similarly .

S is true from the evidence. Suppose that C is true in the state. We will be sampling for R .

The Markov blanket of R is its parents ($\{C\}$), its children ($\{W\}$), and the other parents of its children ($\{S\}$). We use the following distributions to sample R .

$$\begin{aligned} \mathbb{P}(R | MB(R)) &= \mathbb{P}(R | C = t, S = t, W = t) = \alpha \mathbb{P}(R | C = t) \mathbb{P}(W = t | R, S = t) \\ &= \alpha \langle 0.8, 0.2 \rangle \langle 0.99, 0.90 \rangle \\ &= \alpha \langle 0.792, 0.18 \rangle \\ &= \alpha \langle \frac{0.792}{0.972}, \frac{0.18}{0.972} \rangle = \langle \frac{22}{27}, \frac{5}{27} \rangle \end{aligned}$$

$\mathbb{P}(R | \neg C, S, W)$ is calculated similarly.