

Details of the wumpus world calculations

$$\mathbb{P}(P_{1,3} | \text{known}, b) = \alpha' \mathbb{P}(P_{1,3}) \sum_{\text{frontier}} \mathbb{P}(b | \text{known}, P_{1,3}, \text{frontier}) \mathbb{P}(\text{frontier})$$

First, compute with $P_{1,3} = \text{true}$:

Compute the sum over the frontier:

$$\begin{aligned} \mathbb{P}(b | \text{known}, P_{1,3}, P_{2,2}, P_{3,1}) \mathbb{P}(P_{2,2}, P_{3,1}) &= 1 \times 0.2 \times 0.2 \\ &= 0.04 \end{aligned}$$

$$\begin{aligned} \mathbb{P}(b | \text{known}, P_{1,3}, \neg P_{2,2}, P_{3,1}) \mathbb{P}(\neg P_{2,2}, P_{3,1}) &= 1 \times 0.8 \times 0.2 \\ &= 0.16 \end{aligned}$$

$$\begin{aligned} \mathbb{P}(b | \text{known}, P_{1,3}, P_{2,2}, \neg P_{3,1}) \mathbb{P}(P_{2,2}, \neg P_{3,1}) &= 1 \times 0.2 \times 0.8 \\ &= 0.16 \end{aligned}$$

$$\begin{aligned} \mathbb{P}(b | \text{known}, P_{1,3}, \neg P_{2,2}, \neg P_{3,1}) \mathbb{P}(\neg P_{2,2}, \neg P_{3,1}) &= 0 \times 0.8 \times 0.8 \\ &= 0.00 \end{aligned}$$

The sum is: $0.04 + 0.16 + 0.16 + 0.00 = 0.36$

$\mathbb{P}(P_{1,3}) = 0.2$, therefore:

$$\mathbb{P}(P_{1,3}) \sum_{\text{frontier}} \mathbb{P}(b | \text{known}, P_{1,3}, \text{frontier}) \mathbb{P}(\text{frontier}) = 0.2 \times 0.36 = 0.072$$

Then, compute with $P_{1,3} = \text{false}$:

Compute the sum over the frontier:

$$\begin{aligned} \mathbb{P}(b | \text{known}, \neg P_{1,3}, P_{2,2}, P_{3,1}) \mathbb{P}(P_{2,2}, P_{3,1}) &= 1 \times 0.2 \times 0.2 \\ &= 0.04 \end{aligned}$$

$$\begin{aligned} \mathbb{P}(b | \text{known}, \neg P_{1,3}, \neg P_{2,2}, P_{3,1}) \mathbb{P}(\neg P_{2,2}, P_{3,1}) &= 0 \times 0.8 \times 0.2 \\ &= 0.00 \end{aligned}$$

$$\begin{aligned} \mathbb{P}(b | \text{known}, \neg P_{1,3}, P_{2,2}, \neg P_{3,1}) \mathbb{P}(P_{2,2}, \neg P_{3,1}) &= 1 \times 0.2 \times 0.8 \\ &= 0.16 \end{aligned}$$

$$\begin{aligned} \mathbb{P}(b | \text{known}, \neg P_{1,3}, \neg P_{2,2}, \neg P_{3,1}) \mathbb{P}(\neg P_{2,2}, \neg P_{3,1}) &= 0 \times 0.8 \times 0.8 \\ &= 0.00 \end{aligned}$$

The sum is: $0.04 + 0.00 + 0.16 + 0.00 = 0.20$

$\mathbb{P}(\neg P_{1,3}) = 0.8$, therefore:

$$\mathbb{P}(\neg P_{1,3}) \sum_{\text{frontier}} \mathbb{P}(b | \text{known}, \neg P_{1,3}, \text{frontier}) \mathbb{P}(\text{frontier}) = 0.8 \times 0.20 = 0.16$$

$$\mathbb{P}(P_{1,3} | \text{known}, b) = \alpha' \langle 0.072, 0.16 \rangle = \langle \frac{0.072}{0.072+0.16}, \frac{0.16}{0.072+0.16} \rangle = \langle 0.31, 0.69 \rangle$$

$$\underline{\mathbb{P}(P_{1,3} | \text{known}, b) = 0.31} \quad (= \mathbb{P}(P_{3,1} | \text{known}, b) \text{ by symmetry})$$