CS 5811
Nov. 7, 2016

BBN approximate inference techniques

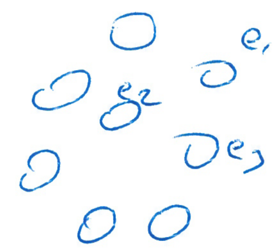
query: $P(x \mid e_1, e_2, e_3)$

1. Rejection sampling

process

1. simulate the network from top to bottom (root)

   obtain N samples



2. reject (throw away) all samples where e_1, e_2, e_3 are not true.

3. among the remaining:

   n_1 has x true

   n_2 has x false

$$p(x \mid e_1, e_2, e_3) = \frac{n_1}{n_2 + n_1} \text{ \} total}$$

entire "world"

Problem: - time wasted on generating not useful samples

- $n_1 + n_2$ can be small (~~if~~ when $P(e_1, e_2, e_3)$ is small.

Suppose that $n_1 + n_2 = 3$

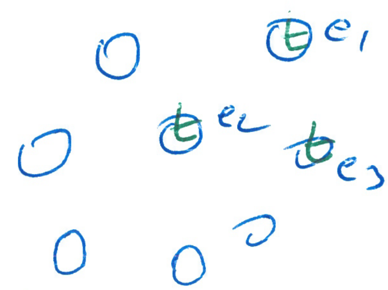
$$p(x \mid e_1, e_2, e_3) = \frac{0}{3}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}$$

2. Likelihood weighting

$p(x \mid e_1, e_2, e_3)$

process

1. simulate the network from the root nodes to the leaf nodes
   the network has the evidence variables ~~set~~ set to the values given



2. obtain N samples

for each sample initialize the weight to 1

start from the root
consider sampling for variable v

if $v \neq e_1$ or e_2 or e_3
sample exactly as in ~~rejection~~ direct sampling
(use the parents' distribution to sample from)
else (v is an evidence variable)
multiply the weight by prob(v | parents)

after sampling :

$$
\begin{array}{cccccc}
 & & & N \text{ samples} & & \\
 & & e_1 & e_2 & e_3 & \\
w_1 & \langle & t & t & t & \rangle \\
w_2 & \langle & t & t & t & \rangle \\
 & \vdots & & & & \\
w_N & \langle & \underbrace{t} & \underbrace{t} & \underbrace{t} & \rangle
\end{array}
$$

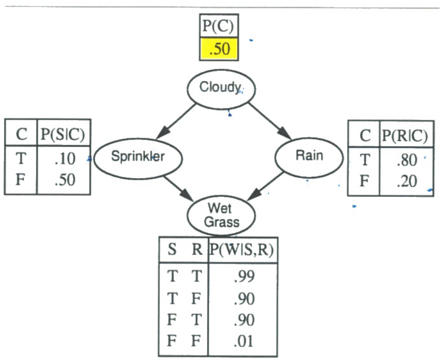
no need for rejections

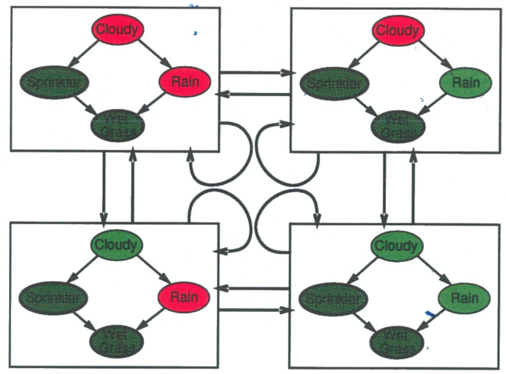
$$
p(x \mid e_1, e_2, e_3) = \frac{\sum w_i \text{ for samples } x = t}{\sum w_i \text{ all samples}}
$$

problems: 1. if the probability of the evidence is low, our weights will be low. If there are a "good" number of evidence variables, the multiplied weights will be low.

2. if our evidence variables are in the lower section of the BBN then their effect will not be seen on the upper variables.

CS5811 handout
The procedure to obtain the sampling distributions for MCMC





<t, t, f, t) randomly assigned

The probability of a variable given its Markov blanket is proportional to the probability of the variable given its parents times the probability of each child given its respective parents:

$$P(x_i'|mb(X_i)) = \alpha P(x_i'|parents(X_i) \times \prod_{Y_j \in children(X_i)} P(y_j|parents(Y_j))$$

Consider the query $\mathbf{P}\,(R|S, W)$. S is true from the evidence. Suppose that R is true in the state.

The Markov blanket of C is its parents ($\emptyset$), its children ($\{R, S\}$), and the other parents of its children ($\emptyset$). We use the following distributions to sample C.

$\mathbf{P}\,(C|MB(C)) = \mathbf{P}\,(C|R, S) = \alpha\ \mathbf{P}\,(C)\ \mathbf{P}\,(S|C)\ \mathbf{P}\,(R|C)$
$= \alpha < 0.5, 0.5 > < 0.1, 0.5 > < 0.8, 0.2 >$
$= \alpha < 0.04, 0.05 >$
$\boxed{= < \frac{4}{9}, \frac{5}{9} >}$

we'll sample for C from this distribution.

For the states where R is false, $\mathbf{P}\,(C|\neg R, S)$ is calculated similarly .

The Markov blanket of R is its parents ($\{C\}$), its children ($\{W\}$), and the other parents of its children ($\{S\}$). We use the following distributions to sample R.

$\mathbf{P}\,(R|MB(R)) = \mathbf{P}\,(R|C, S, W) = \alpha\ \mathbf{P}\,(R|C)\ \mathbf{P}\,(W|R, S)$
$= \alpha < 0.8, 0.2 > < 0.99, 0.90 >$
$= \alpha < 0.792, 0.18 >$
$= \alpha < \frac{0.792}{0.972}, \frac{0.18}{0.972} > = \boxed{< \frac{22}{27}, \frac{5}{27} >}$

$\mathbf{P}\,(R|\neg C, S, W)$ is calculated similarly.