Temporal Constraint Networks

Addition to Chapter 6

Outline

- Temporal reasoning
- Qualitative networks
  - The interval algebra
  - The point algebra
- Quantitative temporal networks

Reference: Chapter 12 of the book titled "Constraint Processing," by Rina Dechter, Morgan Kaufmann Publishers (Elsevier Science), 2003.

Temporal Reasoning

- Temporal objects: points or intervals
- Temporal constraints: qualitative or quantitative
- Temporal knowledge base
- Consistency check routines
- Inference routines
- Query answering mechanisms

Example 1

- Given temporal information
  - Please come before or after lunch
  - I go to lunch before my 1:00 o'clock class
  - Lunch starts at 12:00
  - Lunch takes half an hour to an hour
  - I have class at 11:00
- Derive answers to queries
  - Is it possible that a proposition P holds at time t_1?
  - What are the possible times at which a proposition P holds?
  - What are the possible temporal relationships between two propositions P and Q?

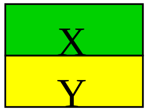
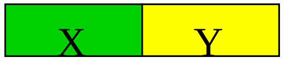
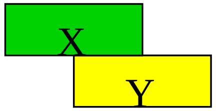
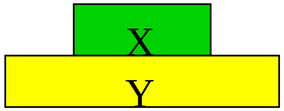
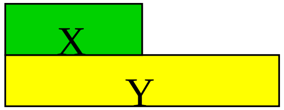
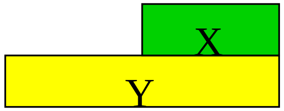
Example 2

John was not in the room when I touched the switch to turn on the light, but John was in the room later when the light went out.

Events:
Switch: time of touching the switch
Light: time the light was on
Room: time that John was in the room

Is this information consistent?
If it is consistent what are the possible scenarios?

Interval algebra (IA)

Relation	Symbol	Inverse	Example
X before Y	b	bi	
X equal Y	=	=	
X meets Y	m	mi	
X overlaps Y	o	oi	
X during Y	d	di	
X starts Y	s	si	
X finishes Y	f	fi	

Representation

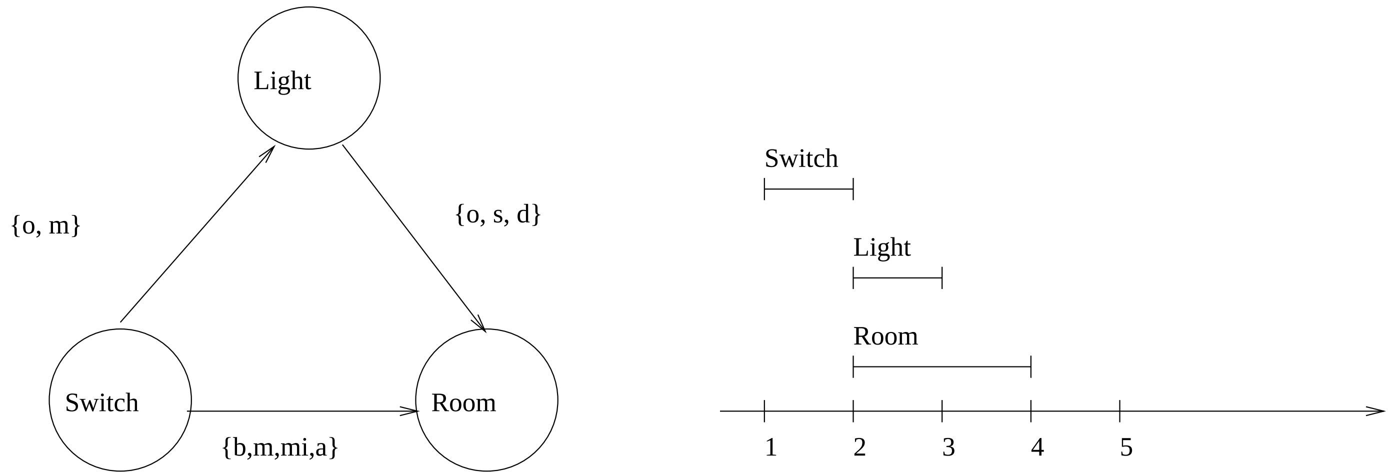
$I\{r_1, \ldots r_k\}\ J$ represents $(I\ r_1\ J) \vee \ldots (I\ r_k\ J)$

For example $I\{s, si, d, di, f, fi, o, oi, =\}\ J$ expresses the fact that intervals I and J intersect (it exludes b, bi, m, mi).

John was not in the room when I touched the switch to turn on the light, but John was in the room later when the light went out.

1. *Switch* $\{o, m\}$ *Light*
2. *Switch* $\{b, m, mi, a\}$ *Room*
3. *Light* $\{o, s, d\}$ *Room*

IA Constraint graph (network)



A *solution* is an assignment of a pair of numbers to each variable such that no constraint is violated

Constraint graph terms

- In a *constraint graph*, the nodes represent the variables and an edge represents a direct constraint (coming from the IA relation set)
- A *universal constraint* permits all relationships between two variables and is represented by the lack of an edge between the variables.
- A constraint C' can be *tighter* than constraint C'', denoted by $C' \subseteq C''$, yielding a partial order between IA networks. A network N'' is tighter than network N' if the partial order $\subseteq$ is satisfied for all the corresponding constraints.
- The *minimal network* of M is the unique equivalent network of M which is minimal with respect to $\subseteq$.

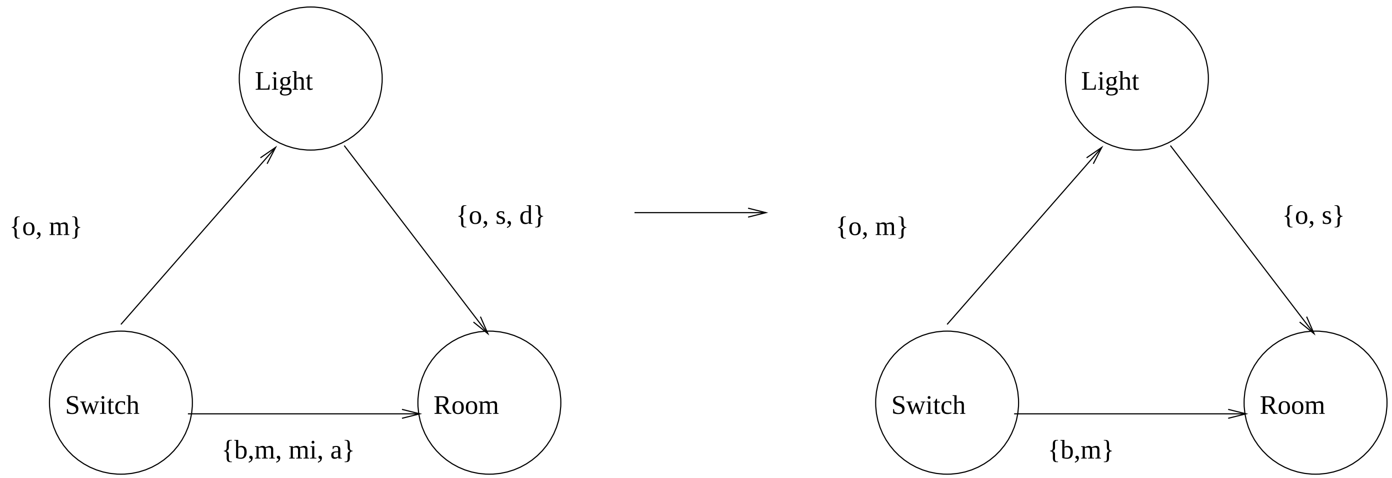
Reasoning tasks for IA networks

- decide consistency
- find one or more solutions
- compute the minimal network

All are generally intractable, so

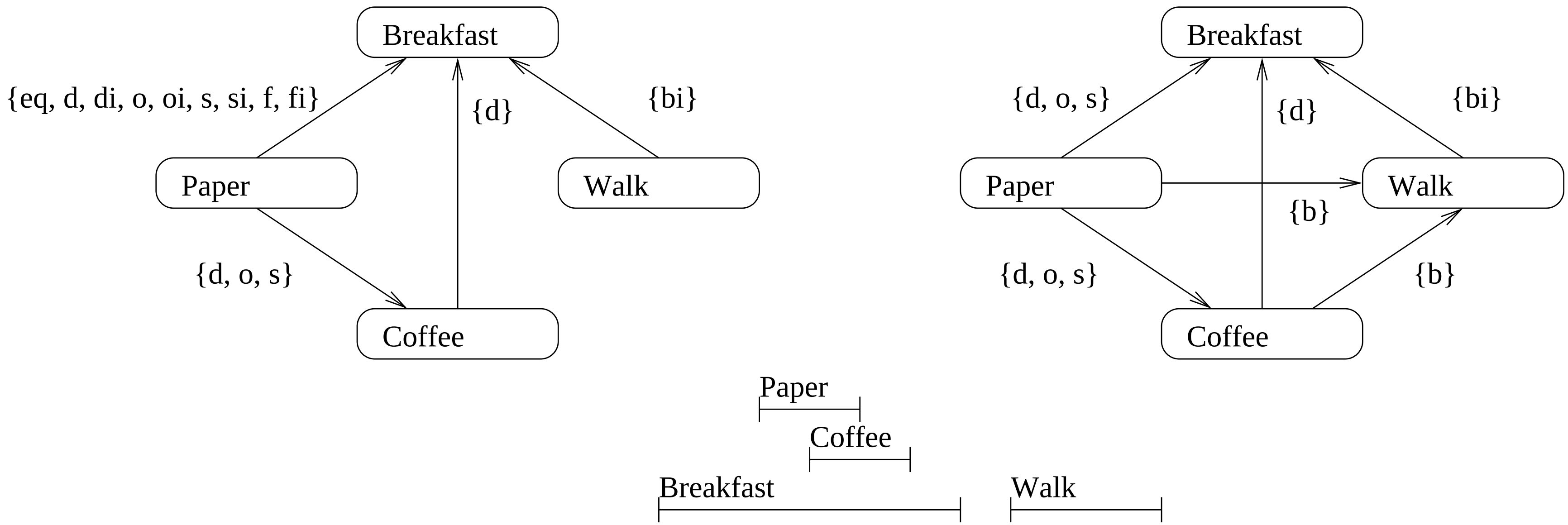
- improve exponential search algorithms such as backtracking, or
- resort to local inference procedures

Minimal network



Example

Fred was reading the paper while eating his breakfast. He put the paper down and drank the last of his coffee. After breakfast he went for a walk.



Path Consistency in CSPs

- Given a constraint network $R = (X, D, C)$, a *two-variable set* $\{x_i, x_j\}$ *is path-consistent* relative to variable x_k iff for every consistent assignment $(< x_i, a_i >, < x_j, a_j >)$ there is a value $a_k \in D_k$ such that the assignment $(< x_i, a_i >, < x_k, a_k >)$ is consistent and $(< x_k, a_k >, < x_j, a_j >)$ is consistent.
- Alternatively, a *binary constraint* R_{ij} *is path-consistent* relative to x_k iff for every pair $(a_i, a_j) \in R_{ij}$ where a_i and a_j are from their respective domains, there is a value $a_k \in D_k$ such that $(a_i, a_k) \in R_{ik}$ and $(a_k, a_j) \in R_{kj}$.

Path-consistency in CSPs (cont'd)

- A *subnetwork over three variables* $\{x_i, x_j, x_k\}$ *is path-consistent* iff for any permutation of (i, j, k), R_{ij} is path-consistent relative to x_k.
- A *network is path-consistent* iff for every R_{ij} (including universal binary relations) and for every $k \neq i, j$, R_{ij} is path-consistent relative to x_k.

Path-consistency in IA

- An IA network is *path-consistent* if for every three variables x_i, x_j, x_k, $C_{ij} \subseteq C_{ik} \otimes C_{kj}$.
- The *intersection of two IA relations* R' and R'', denoted by $R' \oplus R''$, is the set-theoretic intersection $R' \cap R''$.
- The *composition of two IA relations*, $R' \otimes R''$, R' between intervals I and K and R'' between intervals K and J, is a new relation between intervals I and J, induced by R' and R'' as follows.

Composition ($\otimes$)

- The composition of two basic relations r' and r'' is defined by a *transitivity table* (see a portion of it on the next slide).
- The composition of two *composite* relations R' and R'', denoted by $R' \otimes R''$, is the composition of the constituent basic relations:

$$R' \otimes R'' = \{r' \otimes r'' | r' \in R', r'' \in R''\}$$

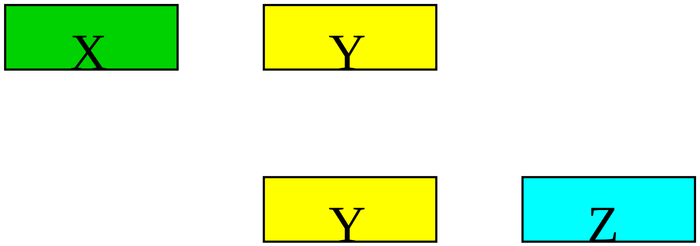
Composition of basic relations

	b	s	d	o	m
b	b	b	b o m d s	b	b
s	b	s	d	b o m	b
d	b	d	d	b o m d s	b
o	b	o	o d s	b o m	b
m	b	m	o d s	b	b

Composition examples

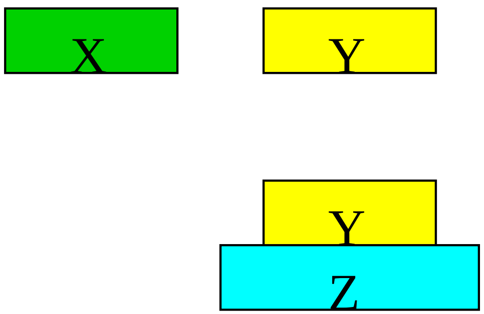
X before Y, Y before Z

X before Z

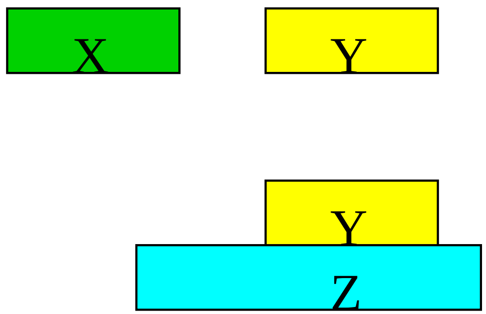


X before Y, Y during Z

X {b, o, m, d, s} Z



before



overlaps

Qualitative Path Consistency (QPC) Algorithm

function QPC-1 (T)
returns a path consistent IA network
  input: T, an IA network with n variables

  repeat
    $S \leftarrow T$
    for $k \leftarrow 1$ to n **do**
      for $i,j \leftarrow 1$ to n **do**
        $C_{ij} \leftarrow C_{ij} \oplus C_{ik} \otimes C_{kj}$
  until $S = T$
  return T

Example

Apply $C_{SR} \leftarrow C_{SR} \oplus (C_{SL} \otimes C_{LR})$

$$C_{SR} \leftarrow \{b, m, i, a\} \oplus (\{o, m\} \otimes \{o, s, d\})$$
$$C_{SR} \leftarrow \{b, m, i, a\} \oplus \{b, o, m, d, s\}$$
$$C_{SR} \leftarrow \{b, m\}$$

$$o \otimes o = b, o, m$$
$$o \otimes s = o$$
$$o \otimes d = o, d, s$$
$$m \otimes o = b$$
$$m \otimes s = m$$
$$m \otimes d = o, d, s$$

Minimizing networks using path-consistency

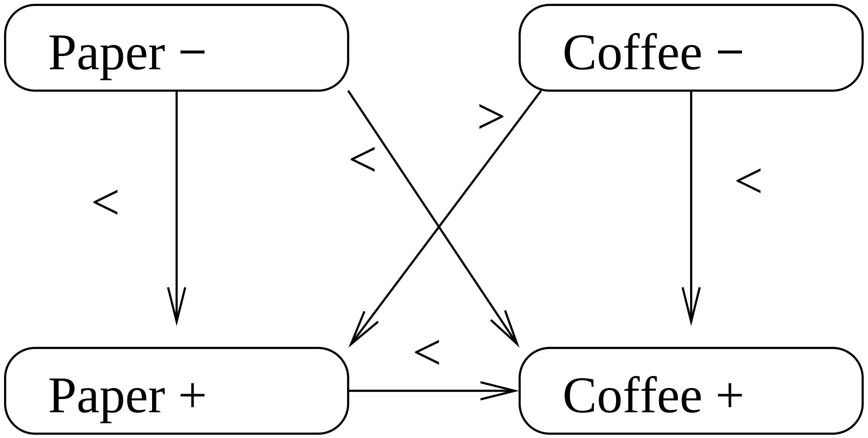
- In some cases, path-consistency algorithms are *exact*—they are guaranteed to generate the minimal network and therefore decide consistency.
- In general, IA networks are NP-complete, backtracking search is needed to generate a solution.
- Even when the minimal network is available, it is not guaranteed to be globally consistent to allow backtrack-free search.
- Path-consistency can be used for forward checking.

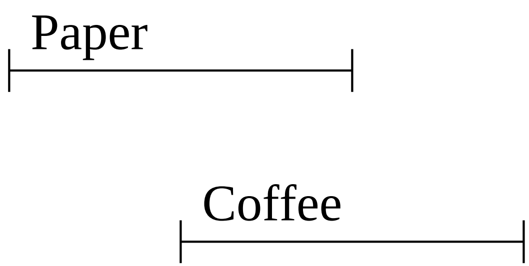
The point algebra (PA)

- It is a model alternative to IA.
- It is less expressive: there are three basic types of constraints between points P and Q: $P < Q$, $P = Q$, $P > Q$.
- Reasoning tasks over PAs are polynomial

Example

Fred put the paper down and drank the last of his coffee.





Examples

- $I\{s,d,f,=\}J$ where $I = [x,y]$ and $J = [z,t]$ can be represented with

$$x < y, z < t, x < t, x \geq z, y \leq t, y > z$$

- However, $I\{b,a\}J$ where $I = [x,y]$ and $J = [z,t]$ cannot be represented with a PA network

Composition in the PA

	<	=	>
<	<	<	?
=	<	=	>
>	?	=	>

"?" expresses the universal relation.

Path consistency

- It is defined using composition and the transitivity table
- Path consistency decides the consistency of a PA network in O(n^3) steps.
- Consistency and solution generation of PA networks can also be accomplished in O(n^2).
- The minimal network of a PA consistent network can be obtained using 4-consistency in O(n^4) steps.
- The minimal network of CPA networks can be obtained by path-consistency in O(n^3).
  Convex PA (CPA) networks have only $\{<, \leq, =, \geq, >\}$ and not $\neq$.

Quantitative Temporal Networks

- Ability to express metric information on duration and timing of events
- John travels to work either by car (30–40 minutes) or by bus (at least 60 minutes). Fred travels to work either by car (20–30 minutes) or in a carpool (40–50 minutes). Today John left home between 7:10 and 7:20A.M., and Fred arrived at work between 8:00 and 8:10A.M. We also know that John arrived at work 10-20 minutes after Fred left home.
- Is the information in the story consistent?
- Is it possible that John took the bus and Fred used the carpool?
- What are the possible times at which Fred left home?

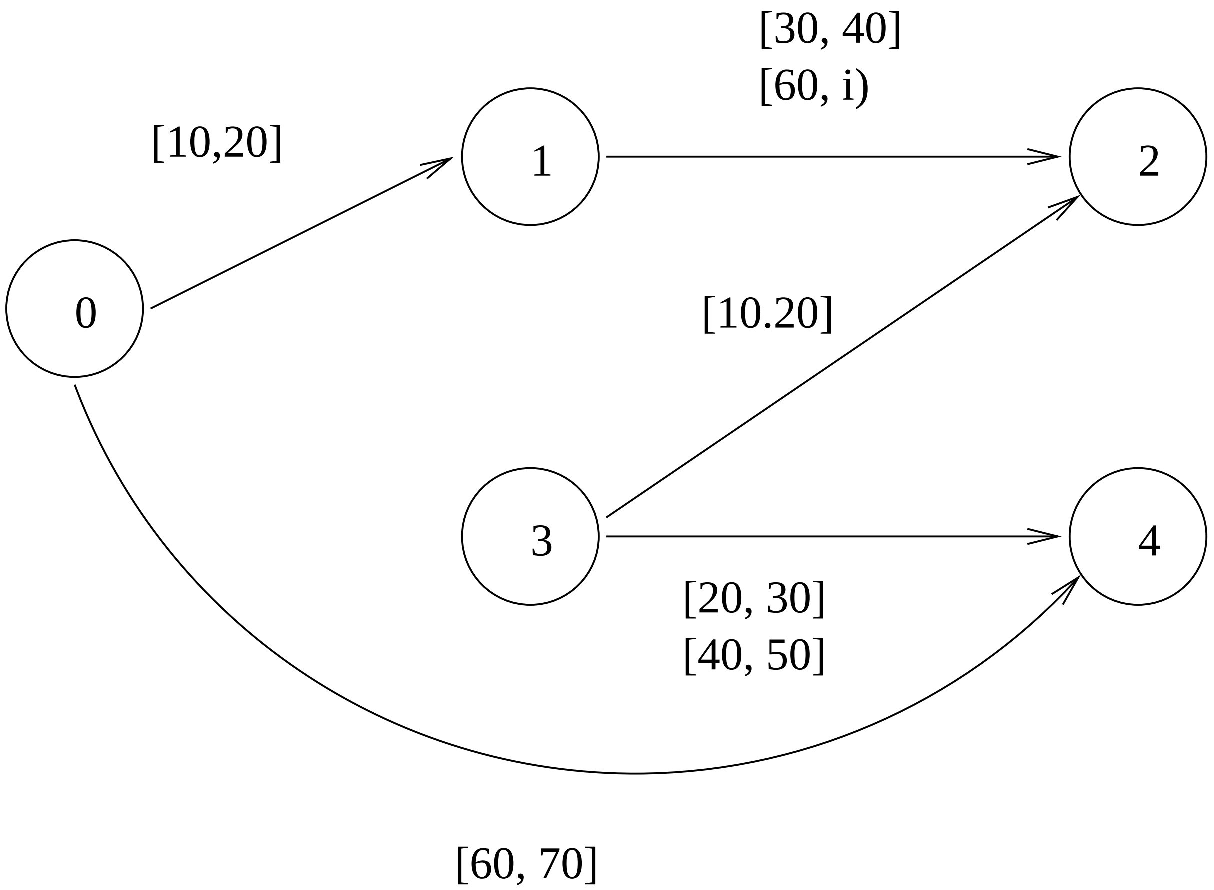
Representation

- Proposition P_1: John was traveling to work ($[x_1, x_2]$)
- Proposition P_2: Fred was traveling to work ($[x_3, x_4]$)
- John travels to work either by car (30–40 minutes) or by bus (at least 60 minutes).
  $30 \leq x_2 - x_1 \leq 40$ or $x_2 - x_1 \geq 60$
- Fred travels to work either by car (20–30 minutes) or in a carpool (40–50 minutes).
  $20 \leq x_4 - x_3 \leq 30$ or $40 \leq x_4 - x_3 \leq 50$

Representation (cont'd)

- Proposition P_1: John was traveling to work ($[x_1, x_2]$)
- Proposition P_2: Fred was traveling to work ($[x_3, x_4]$)
- Today John left home between 7:10 and 7:20A.M. (Assign $x_0 =$ 7:00A.M.)
  $10 \leq x_1 - x_0 \leq 20$
- Fred arrived at work between 8:00 and 8:10A.M.
  $60 \leq x_4 - x_0 \leq 70$
- John arrived at work 10-20 minutes after Fred left home.
  $10 \leq x_4 - x_0 \leq 20$

The constraint graph



Temporal Constraint Satisfaction Problem (TCSP)

A *temporal constraint satisfaction problem (TCSP)* involves a set of variables $\{x_1, \ldots, x_n\}$ having continuous domains; each variable represents a time point. Each constraint is represented by a set of intervals $\{I_1, \ldots I_k\} = \{[a_1, b_1], \ldots, [a_k, b_k]\}$.

A unary constraint T_i restricts the domain of a variable x_i to the given set of intervals; that is, it represents the disjunction

$$(a_1 \leq x_i \leq b_1) \vee \ldots \vee (a_k \leq x_i \leq b_k)$$

A binary constraint T_{ij} constrains the permissible values for the distance $x_j - x_i$; it represents the disjunction

$$(a_1 \leq x_j - x_i \leq b_1) \vee \ldots \vee (a_k \leq x_j - x_i \leq b_k)$$

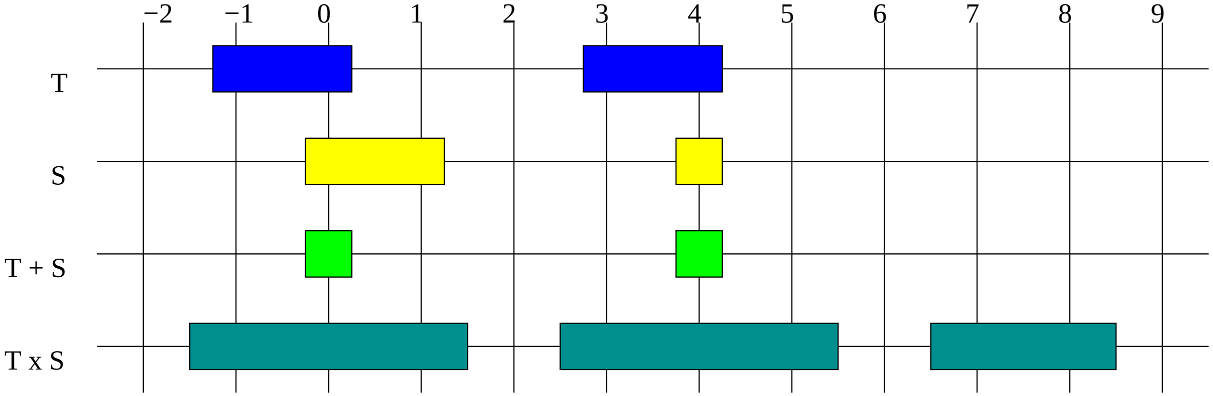
TCSP (cont'd)

- Assume that constraints are given in a *canonical form* in which all intervals are pair-wise disjoint.
- A special time point, x_0, represents the "beginning of the world." Each unary constraint can be represented as a binary constraint relative to x_0.
- A tuple $x = \{a_1, \ldots, a_n\}$ is called a *solution* if the assignment $\{x_1 = a_1, \ldots, x_n = a_n\}$ does not violate any constraint.

Minimal and binary decomposable networks

- Given a TCSP, a value v is a *feasible value* for variable x_i if there exists a solution in which $x_i = v$.
- The set of all feasible values of a variable is called the *minimal domain*.
- A minimal constraint T_{ij} between x_i and x_j is the set of all feasible values for $x_i - x_j$.
- A network is minimal iff its domains and constraints are minimal.
- A network is *binary decomposable* if every consistent assignment of values to a set of variables S can be extended to a solution.

Binary operators on constraints



$$
\begin{aligned}
T &= \{[-1.25, 0.25]\}, [2.75, 4.25]\} \\
S &= \{[-0.25, 1.25]\}, [3.75, 4.25]\} \\
T \oplus S &= \{[-0.25, 0.25]\}, [3.75, 4.25]\} \\
T \otimes S &= \{[-1.50, 1.50], [2.50, 5.50], [6.50, 8.50]\}
\end{aligned}
$$

Binary operators on constraints (cont'd)

Let $T = \{I_1, \ldots, I_l\}$ and $S = \{J_1, \ldots, J_m\}$ be two constaints. Each is a set of intervals of a temporal variable or a temporal binary constraint.

- The union of T and S, denoted by $T \cup S$, only admits values that are allowed by either T or S, that is, $T \cup S$ = $\{I_1, \ldots, I_l, J_1, \ldots, J_m\}$.
- The intersection of T and S, denoted by $T \oplus S$, admits only values that are allowed by both T and S, that is, $T \oplus S = \{K_1, \ldots, K_n\}$ where $K_k = I_i \cap J_j$ for some i and j. Note that $n \leq l + m$.

Binary operators on constraints (cont'd)

- The composition of T and S, denoted by $T \otimes S$, admits only values r for which there exist $t \in T$ and $s \in S$, such that $t + s = r$, that is $T \otimes S = \{K_1, \ldots K_n\}$, where $K_k = [a + c, b + d]$ for some $I_i = [a, b]$, and $J_j = [c, d]$. Note that $n \leq l \times m$.

Simple temporal problems (STPs)

- It is a subclass of TCSPs where all constraints specify a single interval (no disjunctions).
- Each edge $i \rightarrow j$ is labeled by a single interval $[a_{ij}, b_{ij}]$ that represents the constraint

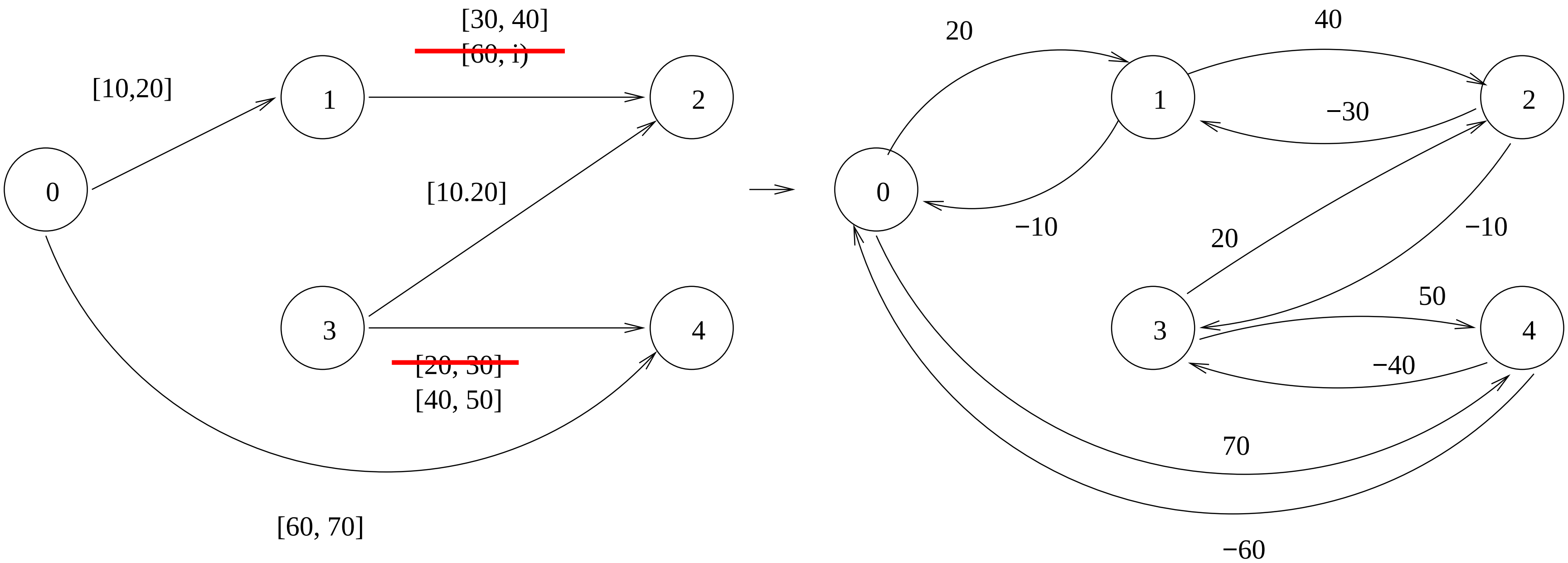
$$a_{ij} \leq x_j - x_i \leq b_{ij}$$

or

$$x_j - x_i \leq b_{ij} \text{ and } x_i - x_j \leq -a_{ij}$$

- Can be represented and solved as a system of linear inequalities but a better graph algorithm exists: first convert the graph into a *distance graph*

Distance graph example



Distance graph

- An STP can be associated with a directed-edge weighted graph $G_d = (v, E_d)$, called the *distance graph*. It has the same node set as G, and each edge $i \rightarrow j \in E_d$ is labeled by a weight a_{ij} representing the linear inequality $x_j - x_i \leq a_{ij}$.
- Each path from i to j in G_d, $i_0 = i, i_1, \ldots i_k = j$, induces the following constraint on the distance $x_j - x_i$:

$$x_j - x_i \leq \sum_{j=1}^{k} a_{i_{j-1}, i_j}$$

Distance graph (cont'd)

- If there is more than one path from i to j, then it can easily be verified that the intersection of all the induced path constraints yields
$$x_j - x_i \leq d_{ij}$$
where d_{ij} is the length of the shortest path from i to j.
- Theorem: An STP T is consistent iff its distance graph G_d has no negative cycles
- For any pair of connected nodes i and j, the shortest paths satisfy $d_{oj} \leq d_{oi} + a_{ij}$; thus,
$$d_{oj} - d_{oi} \leq a_{ij}$$

Distance graph (cont'd)

- Let G_d be the distance graph of a consistent STP. Two consistent scenarios are given by

$$S_1 = (d_{01}, \ldots, d_{0n}) \text{ and } S_2 = (-d_{10}, \ldots, -d_{n0})$$

which assign to each variable its latest and earliest possible times, respectively.

- A given STP can be effectively specified by a complete directed graph, called *d-graph*, where each edge is labeled by the shortest-path length d_{ij} in G_d.
- Decomposability theorem: Any consistent STP is backtrack-free (decomposable) relative to the constraints in its *d*-graph.

Lengths of shortest paths (d_{ij})

	0	1	2	3	4
0	0	20	50	30	70
1	-10	0	40	20	60
2	-40	-30	0	-10	30
3	-20	-10	20	0	50
4	-60	-50	-20	-40	0

The minimal network

	0	1	2	3	4
0	[0]	[10,20]	[40,50]	[20,30]	[60,70]
1	[-20,-10]	[0]	[30,40]	[10,20]	[50,60]
2	[-50,-40]	[-40,-30]	[0]	[-20,-10]	[20,30]
3	[-30,-20]	[-20,-10]	[10,20]	[0]	[40,50]
4	[-70,-60]	[-60,-50]	[-20,-30]	[-50,-40]	[0]

Floyd-Warshall's Algorithm (apsp)

function ALL-PAIRS-SHORTEST-PATS (*G*)
returns a *d*-graph
input: Distance graph *Gi* = *(V,E)* with weights a_{ij} for $(i, j) \in E$.

for *i* $\leftarrow$ 1 to *n* **do**
$d_{ii} \leftarrow 0$
for *i,j* $\leftarrow$ 1 to *n* **do**
$d_{ij} \leftarrow a_{ij}$
for *k* $\leftarrow$ 1 to *n* **do**
for *i,j* $\leftarrow$ 1 to *n* **do**
$d_{ij} \leftarrow \min \{d_{ij}, d_{ik} + d_{kj}\}$

Summary

- Floyd-Warshall's algorithm runs in O(n^3) and detects negative cycles simply by examining the sign of the diagonal elements d_{ii}.
- Once the *d*-graph is available, assembling a solution takes only O(n^2) time, because each successive assignment only needs to be checked against previous assignments and is guaranteed to remain unaltered.
- Thus, finding a solution takes O(n^3) time.
- Note that in TCSP, path consistency can be checked in polynomial time but does not guarantee minimality.

Summary (cont'd)

- Any constraint network in PA is a special case of a TCSP lacking metric information.
- A PA can be translated into a TCSP in a straightforward manner.
  - $x_j < x_i$ translates to $T_{ij} = \{(-\infty, 0)\}$
  - $x_j \leq x_i$ translates to $T_{ij} = \{(-\infty, 0]\}$
  - $x_j = x_i$ translates to $T_{ij} = \{[0]\}$
  - $x_j \neq x_i$ translates to $T_{ij} = \{(-\infty, 0), (0, \infty)\}$
- IA networks cannot always be translated into binary TCSPs because such a translation may require nonbinary constraints:
  X {b, bi} $Y \equiv X_e < Y_s \vee Y_e < X_s$

Example: Autominder

- To assist people with memory impairment.
- Model their daily activities, including temporal constraints on their performance
- Monitor the execution of those activities
- Decide whether and when to issue reminders

Example: Autominder (cont'd)

ACTION	TARGET TIME
Start laundry	Before 10 a.m.
Put clothes in dryer	Within 20 minutes of washer ending
Fold clothes	Within 20 minutes of dryer ending
Prepare lunch	Between 11:45 and 12:15
Eat lunch	At end of prepare lunch
Check pulse	Between 11:00 and 1:00, and between 3:00 and 5:00
depending on pulse take medication	at end of check pulse

Other examples

- US NINDS (National Institute of Neurological Disorders and Stroke) guidelines for treatment of potential stroke (thrombolytic) patient
  - hospital door to doctor: 10 minutes
  - door to neurological expert: 15 minutes
  - door to CT scan completion: 25 minutes
  - . . .
- Space facility crew activity planning
- Control of spacecraft on another planet