## CS5811 Simple Temporal Network Example

Consider the following events with associated time intervals:

1. I was in Houghton at 8:30.
2. I left home between $8: 05$ and $8: 10$.
3. It takes me 20 minutes to drive to the bridge.
4. I waited 5-10 minutes at the bridge.

Given the above constraints, the only scenario that is possible is that I left at 8:05 and waited only 5 minutes at the bridge. In order to implement an algorithm that performs a similar type of reasoning, we first represent in problem as a quantitative constraint graph.

The following is the constraint graph representing the above problem. In this graph, the nodes correspond to events and the arcs show the temporal constraints between two events. " 0 " refers to time zero (8:00 in this example). "L" refers to the time of leaving home. "B" refers to the time of arriving at the bridge. " H " refers to the time of arriving in Houghton. The first number in each bracket is the lower bound, and the second number is the upper bound for the duration of the event. For example, it took between 5 to 10 minutes after 8:00 to leave home.

$[30,30]$

We then convert the constraint graph into a distance graph. In a distance graph, the semantics of the nodes remain the same. Each constraint arc from A to B is replaced with two arcs. The first arc goes from $A$ to $B$ and is labeled with the upper bound. The second arc goes from $B$ to $A$ and is labeled with the negative of the lower bound. This way, when we are going from "left to right" we are adding the upper bonds; and when we are going form "right to left" we are adding the lower bounds.


The original constraint matrix is as follows. We are using " 99 " as a large number to represent $\infty$, i.e., no constraints.

|  | 0 | 1 | 2 | 3 |
| ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | 10 | 99 | 30 |
| 1 | -5 | 0 | 20 | 99 |
| 2 | 99 | -20 | 0 | 10 |
| 3 | -30 | 99 | -5 | 0 |

The constraint graph after running Floyd-Warshall's algorithm is:

|  | 0 | 1 | 2 | 3 |
| ---: | ---: | ---: | ---: | ---: |
| 0 | 0 | $\mathbf{5}$ | 25 | 30 |
| 1 | -5 | 0 | 20 | 25 |
| 2 | -25 | -20 | 0 | $\mathbf{5}$ |
| 3 | -30 | -25 | -5 | 0 |

Note that the 10 s at $(0,1)$ and $(2,3)$ changed to 5 s . Also, $(0,2),(1,3),(2,0)$, and $(3,1)$ changed from 99.

Example shortest path: from 0 to 1 the direct link is 10 . If we go from 0 to 3 , and then backwards, it is $30-5-20=5$. Similar for from 2 to 3 .

