Chapter 3 Solving Problems By Searching
3.1 –3.4 Uninformed search strategies

CS4811 - Artificial Intelligence

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Outline

Problem-solving agents

Problem formulation

Basic search algorithms
  Tree search
  Graph search

Evaluating search strategies

Uninformed search strategies
  Breadth-first search
  Uniform-cost search
  Depth-first search
  Depth-limited search
  Iterative deepening search
  Bidirectional search
Problem-solving agents

function Simple-Problem-Solving-Agent (*percept*)
returns an action

inputs: *percept*, a percept

private: *seq*, an action sequence, initially empty

*state*, some description of the current world state

*goal*, a goal, initially null

*problem*, a problem formulation

\[
\text{state} \leftarrow \text{Update-State} \left( \text{state}, *percept* \right)
\]

if seq is empty then

\[
\text{goal} \leftarrow \text{Formulate-Goal} \left( \text{state} \right)
\]

\[
\text{problem} \leftarrow \text{Formulate-Problem} \left( \text{state}, \text{goal} \right)
\]

\[
\text{seq} \leftarrow \text{Search} \left( \text{problem} \right)
\]

if seq = failure then return a null action

\[
\text{action} \leftarrow \text{First} \left( \text{seq} \right)
\]

\[
\text{seq} \leftarrow \text{Rest} \left( \text{seq} \right)
\]

return action
Assumptions

- **Static**: The world does not change unless the agent changes it.
- **Observable**: Every aspect of the world state can be seen.
- **Discrete**: Has distinct states as opposed to continuously flowing time.
- **Deterministic**: There is no element of chance.

This is a restricted form of a general agent called *offline* problem solving. The solution is executed “eyes closed.”

*Online* problem solving involves acting without complete knowledge.
Example: Traveling in Romania

- On holiday in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

- **Formulate goal:**
  be in Bucharest

- **Formulate problem:**
  states: various cities
  actions: drive between cities

- **Find solution:**
  sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
  (any solution or optimal solution?)
Distances between cities in Romania
Infrastructure for search algorithms

- A **problem** is defined by five components:
  - **initial state** e.g., “In(Arad)”
  - **actions**, ACTIONS(s) returns the actions applicable in s. e.g., In Arad, the applicable actions are \{Go(Sibiu), Go(Timisoara), Go(Zerind)\}
  - **transition model**, RESULT(s, a) returns the state that results from executing action a in state s e.g., RESULT(In(Arad), Go(Zerind)) = In(Zerind).
  - **goal test**, can be
    - **explicit**, e.g., \(x = \text{“In Bucharest”}\)
    - **implicit**, e.g., \(x = \text{“In a city with an international airport”}\)
  - **path cost** (additive)
    e.g., sum of distances, number of actions executed, etc. \(c(x, a, y)\) is the **step cost** of executing action a in state x and arriving at state y, assumed to be \(\geq 0\)
- A **solution** is a sequence of actions leading from the initial state to a goal state
Selecting a state space

- The real world is absurdly complex
  ⇒ state space must be *abstracted* for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
e.g., “Arad → Zerind” represents a complex set of possible routes, detours, rest stops, etc.
  For guaranteed realizability, any real state “in Arad” must get to some real state “in Zerind”
- (Abstract) solution = set of real paths that are solutions in the real world
- Each abstract action should be “easier” than the original problem!
- Find an abstraction that is *valid* and *useful*. 
Example: The 8-puzzle

Start State

Goal State
Example: The 8-puzzle (cont’d)

- **states**: integer locations of tiles (ignore intermediate positions)
- **actions**: move blank left, right, up, down (ignore unjamming etc.)
- **goal test**: = goal state (given)
- **path cost**: 1 per move
- Note that the optimal solution of $n$-Puzzle family is NP-hard
Basic idea:
offline, simulated exploration of state space
by generating successors of the states that haven’t been explored
(a.k.a. *expanding* states)
function TREE-SEARCH (problem, strategy)

returns a solution, or failure

initialize the frontier using the initial state of problem

loop do

if the frontier is empty then return failure

choose a leaf node and remove it from the frontier

if the node contains a goal state

then return the corresponding solution

then expand the chosen node and add the resulting nodes to the frontier

end
Tree search example
Tree search example
Tree search example
Implementation: states vs. nodes

- A *state* is a (representation of) a physical configuration.
- A *node* is a data structure constituting part of a search tree.
- A node includes: *parent*, *children*, *depth*, *path cost* $g(x)$.
- States do not have parents, children, depth, or path cost!
- The **Expand** function creates new nodes, filling in the various fields and using the **SuccessorFn** of the problem to create the corresponding states.
Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!
Graph search algorithms

Basic idea:
similar to tree-search
keep a separate list of “explored” states
function \textsc{Graph-Search} (\textit{problem})
returns a solution, or failure

initialize the frontier using the initial state of \textit{problem} → initialize the explored set to be empty

\textbf{loop do}
\hspace{1em} if the frontier is empty then return failure
\hspace{1em} choose a leaf node and remove it from the frontier
\hspace{1em} if the node contains a goal state
\hspace{1.5em} then return the corresponding solution
\hspace{1em} add the node to the explored set
\hspace{1em} expand the chosen node and add the resulting nodes to the frontier only if not in the frontier or explored set
\textbf{end}

Note: A $\rightarrow$ shows the lines that are added to the tree search algorithm.
Evaluating search strategies

- A strategy is defined by picking the order of node expansion.
- Strategies are evaluated along the following dimensions:
  - **completeness**—does it always find a solution if one exists?
  - **time complexity**—number of nodes generated/expanded
  - **space complexity**—maximum number of nodes in memory
  - **optimality**—does it always find a least-cost solution?
- Time and space complexity are measured in terms of:
  - $b$ — maximum branching factor of the search tree
  - $d$ — depth of the least-cost solution
  - $m$ — maximum depth of the state space
    (may be $\infty$)
**Uninformed search strategies**

*Uninformed* strategies use only the information available in the problem definition

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search
- Bidirectional search
Breadth-first search

- Expand the shallowest unexpanded node
- Implementation: *frontier* is a FIFO queue, i.e., new successors go at end
Progress of breadth-first search

Breadth-first search on a simple binary tree. At each stage, the node to be expanded next is indicated by a marker. The nodes that are already explored are gray. The nodes with dashed lines are not generated yet.
Progress of breadth-first search

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Breadth-first search on a simple binary tree. At each stage, the node to be expanded next is indicated by a marker. The nodes that are already explored are gray. The nodes with dashed lines are not generated yet.
Properties of breadth-first search

- **Complete:** Yes (if $b$ is finite)
- **Time:** $b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., number of nodes generated is exponential in $d$
- **Space:** $O(b^{d+1})$ (keeps every node in memory)
- **Optimal:** Yes (if cost = 1 per step)

Space is the big problem; can easily generate nodes at 100MB/sec so 24hrs = 8604GB.
Breadth-first search algorithm

function **Breadth-First-Search** (*problem*)
returns a solution, or failure

\[ node \leftarrow \text{a node with } \text{State}=\text{problem.Initial-State}, \text{Path-Cost}=0 \]

if \text{problem.Goal-Test}(node.State) then return Solution(node)

frontier \leftarrow \text{a FIFO queue with node as the only element}

explored \leftarrow \text{an empty set}

loop do

if Empty?(frontier) then return failure

node \leftarrow \text{Pop(frontier) /* chooses the shallowest node in frontier */}

add node.State to explored

for each action in problem.Actions(node.State) do

child \leftarrow \text{Child-Node (problem,node, action)}

if child.State is not in explored or frontier then

if problem.Goal-Test(child.State) then

return Solution(child)

frontier \leftarrow \text{Insert (child, frontier)}
Uniform-cost search

- Expand the least-cost unexpanded node
- Implementation: *frontier* is a queue ordered by path cost
- Equivalent to breadth-first if step costs are all equal
Properties of uniform-cost search

- **Complete:** Yes, if step cost $\geq \epsilon$
- **Time:** # of nodes with $g \leq$ cost of optimal solution, $O(b^{1+[C^*/\epsilon]})$
  where $C^*$ is the cost of the optimal solution
- **Space:** # of nodes with $g \leq$ cost of optimal solution, $O(b^{1+[C^*/\epsilon]})$
- **Optimal:** Yes—nodes expanded in increasing order of $g(n)$
function Uniform-Cost-Search (problem)
returns a solution, or failure
node ← a node with State=problem.Initial-State, Path-Cost = 0
if problem.Goal-Test(node.State) then return Solution(node)
frontier ← a priority ordered by Path-Cost, with node as the only element
explored ← an empty set
loop do
    if Empty?(frontier) then return failure
    node ← pop(frontier) /* chooses the lowest-cost node in frontier */
    add node.State to explored
    for each action in problem.Actions(node.State) do
        child ← Child-Node (problem,node, action)
        if child.State is not in explored or frontier then
            frontier ← Insert (child, frontier)
        else if child.State is in frontier with higher Path-Cost then
            replace that frontier node with child
Depth-first search

- Expand deepest unexpanded node
- Implementation: *frontier* is a LIFO queue, i.e., put successors at front
Progress of depth-first search
Progress of depth-first search
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Progress of depth-first search
Properties of depth-first search

- **Complete:** No: fails in infinite-depth spaces, spaces with loops
  Modify to avoid repeated states along path
  ⇒ complete in finite spaces

- **Time:** $O(b^m)$: terrible if $m$ is much larger than $d$
  but if solutions are dense, may be much faster than breadth-first

- **Space:** $O(bm)$, i.e., linear space!

- **Optimal:** No
Depth-limited search

- It is equivalent to depth-first search with depth limit \( l \), i.e., nodes at depth \( l \) have no successors
- implementation: a recursive implementation is shown on the next page
Properties of depth-limited search

- **Complete**: No (similar to DFS)
- **Time**: $O(b^l)$, where $l$ is the depth-limit
- **Space**: $O(bl)$, i.e., linear space (similar to DFS)
- **Optimal**: No
**Depth-limited search**

```plaintext
function Depth-Limited-Search (problem, limit)
returns a solution, or failure/cutoff
return Recursive-DLS(Make-Node( problem.Initial-State), problem, limit)

function Recursive-DLS (node, problem, limit)
returns a solution, or failure/cutoff
  if problem.Goal-Test(node.State) then return Solution(node)
  else if limit = 0 then return cutoff
  else
    cutoff-occurred? ← false
    for each action in problem.Actions(node.State) do
      child ← Child-Node (problem,node, action)
      result ← Recursive-DLS (child, problem,limit-1)
      if result = cutoff then cutoff-occurred? ← true
      else if result ≠ failure then return result
      if cutoff-occurred? then return cutoff else return failure
```
Iterative deepening search

- Do iterations of depth-limited search starting with a limit of 0. If you fail to find a goal with a particular depth limit, increment it and continue with the iterations.
- Terminate when a solution is found or if the depth-limited search returns *failure*, meaning that no solution exists.
- Combines the linear space complexity of DFS with the completeness property of BFS.
Iterative deepening search \((l = 0)\)
Iterative deepening search ($l = 1$)
Iterative deepening search ($l = 2$)
Iterative deepening search ($l = 3$)
Properties of iterative deepening search

- Complete: Yes
- Time: \( db^1 + (d - 1)b^2 + \ldots + b^d = O(b^d) \)
- Space: \( O(bd) \)
- Optimal: Yes, if step cost = 1
  
  Can be modified to explore uniform-cost tree
Iterative deepening search

function Iterative-Deepening-Search(problem) returns a solution, or failure

for depth ← 0 to ∞ do
    result ← Depth-Limited-Search (problem, depth)
    if result ≠ cutoff then return result
Compare IDS and BFS

Numerical comparison of the number of nodes generated for $b = 10$ and $d = 5$, solution at the far right leaf:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000$$
$$= 123,450$$

$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990$$
$$= 1,111,100$$

IDS does better because other nodes at depth $d$ are not expanded. BFS can be modified to apply the goal test when a node is generated (rather than expanded).
## Summary of algorithms

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</tr>
</thead>
<tbody>
<tr>
<td>Complete?</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Time</td>
<td>$O(b^{d+1})$</td>
<td>$O(b^{1+\lceil C^*/\epsilon \rceil})$</td>
<td>$O(b^m)$</td>
<td>$O(b^l)$</td>
<td>$O(b^d)$</td>
</tr>
<tr>
<td>Space</td>
<td>$O(b^{d+1})$</td>
<td>$O(b^{1+\lceil C^*/\epsilon \rceil})$</td>
<td>$O(bm)$</td>
<td>$O(bl)$</td>
<td>$O(bd)$</td>
</tr>
<tr>
<td>Optimal?</td>
<td>Yes*</td>
<td>Yes*</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Bidirectional search

- Run two simultaneous states:
  one forward from the initial state
  one backward from the goal state

- Motivation: \( b^{d/2} + b^{d/2} \) is much less than \( b^d \)

- Implementation: Replace the goal check with a check to see whether the frontiers of the searches intersect
Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored.

There are a variety of uninformed search strategies available.

Iterative deepening search uses only linear space and not much more time than other uninformed algorithms.
Sources for the slides

- AIMA textbook (3rd edition)
- AIMA slides (http://aima.cs.berkeley.edu/)