Chapter 3 Solving Problems By Searching
3.1 –3.4 Uninformed search strategies

CS4811 - Artificial Intelligence

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Outline

Problem-solving agents

```
function Simple-Problem-Solving-Agent (percept)
returns an action
    inputs: percept, a percept
    private: seq, an action sequence, initially empty
             state, some description of the current world state
             goal, a goal, initially null
             problem, a problem formulation

    state ← Update-State (state,percept)
    if seq is empty then
        goal ← Formulate-Goal (state)
        problem ← Formulate-Problem (state, goal)
        seq ← Search (problem)
    if  seq = failure then return a null action
    action ← First (seq)
    seq ← Rest (seq)
    return action
```

Assumptions

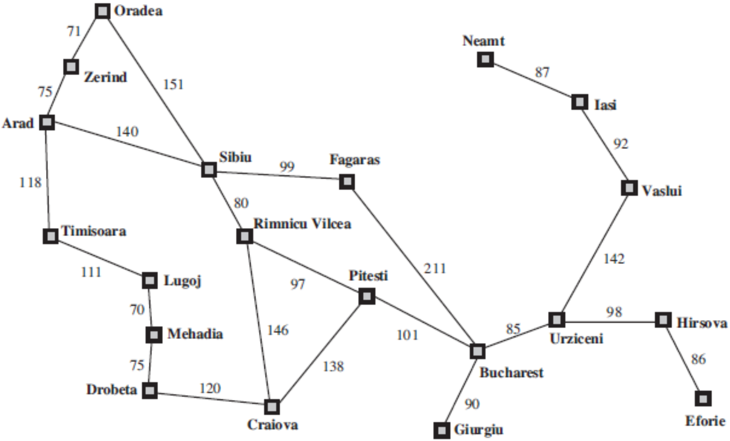
- *Static:* The world does not change unless the agent changes it.
- *Observable:* Every aspect of the world state can be seen.
- *Discrete:* Has distinct states as opposed to continuously flowing time.
- *Deterministic:* There is no element of chance.

This is a restricted form of a general agent called *offline* problem solving. The solution is executed "eyes closed."
Online problem solving involves acting without complete knowledge

Example: Traveling in Romania

- On holiday in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest
- **Formulate goal:**
  be in Bucharest
- **Formulate problem**:
  states: various cities
  actions: drive between cities
- **Find solution**:
  sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
  (any solution or optimal solution?)

Distances between cities in Romania



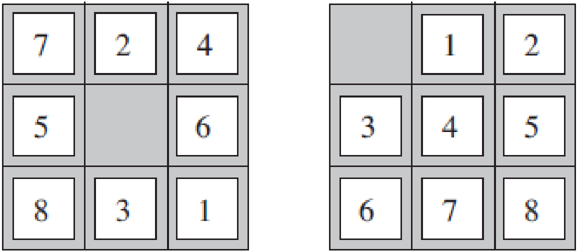
Infrastructure for search algorithms

- A *problem* is defined by five components:
    - *initial state* e.g., "In(Arad)"
    - *actions*, $\textsc{Actions}(s)$ returns the actions applicable in s.
      e.g, In Arad, the applicable actions are
      {Go(Sibiu), Go(Timisoara), Go(Zerind)}
    - *transition model*, $\textsc{Result}(s, a)$ returns the state that results from executing action a in state s
      e.g., $\textsc{Result}$(In(Arad), Go(Zerind)) = In(Zerind).
    - *goal test*, can be
      explicit, e.g., $x =$ "In Bucharest"
      implicit, e.g., $x =$ "In a city with an international airport"
    - *path cost* (additive)
      e.g., sum of distances, number of actions executed, etc.
      $c(x, a, y)$ is the *step cost* of executing action a in state x and arriving at state y, assumed to be ≥ 0
- A *solution* is a sequence of actions leading from the initial state to a goal state

Selecting a state space

- The real world is absurdly complex
  ⇒ state space must be *abstracted* for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  e.g., "Arad → Zerind" represents a complex set
  of possible routes, detours, rest stops, etc.
  For guaranteed realizability, any real state "in Arad" must get
  to some real state "in Zerind"
- (Abstract) solution =
  set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem!
- Find an abstraction that is *valid* and *useful*.

Example: The 8-puzzle



Start State

Goal State

Example: The 8-puzzle (cont'd)

- **states**: integer locations of tiles
  (ignore intermediate positions)
- **actions**: move blank left, right, up, down
  (ignore unjamming etc.)
- **goal test**: = goal state (given)
- **path cost**: 1 per move
- Note that the optimal solution of *n*-Puzzle family is NP-hard

Tree search algorithms

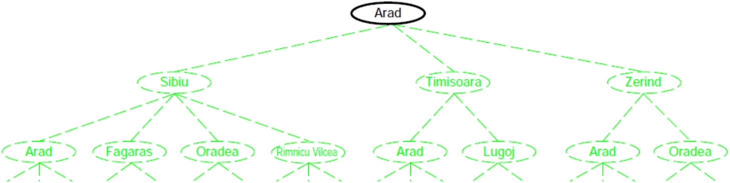
Basic idea:
offline, simulated exploration of state space
by generating successors of the states that haven't been explored
(a.k.a. *expanding* states)

Tree search algorithms (cont'd)

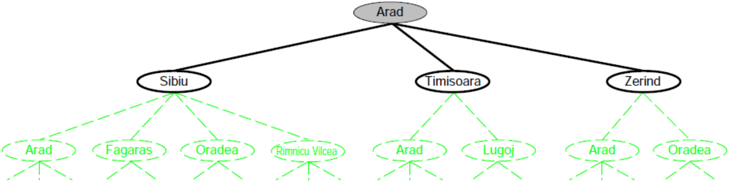
```
function TREE-SEARCH (problem, strategy)
returns a solution, or failure

    initialize the frontier using the initial state of problem
    loop do
        if the frontier is empty then return failure
        choose a leaf node and remove it from the frontier
        if the node contains a goal state
            then return the corresponding solution
        expand the chosen node and add the resulting nodes to the frontier
    end
```

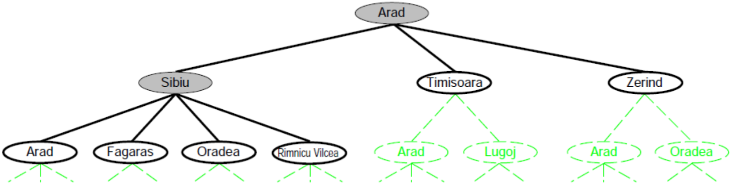
Tree search example



Tree search example

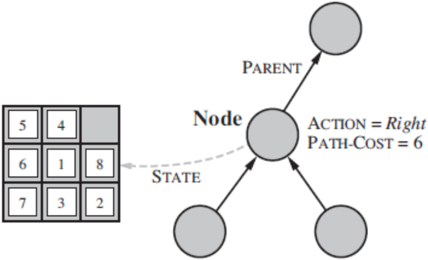


Tree search example



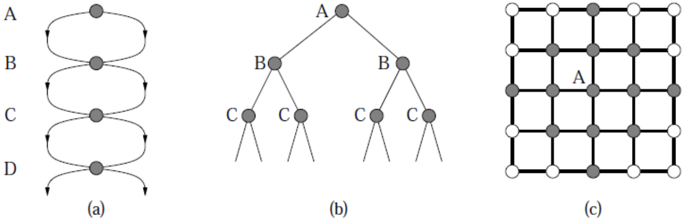
Implementation: states vs. nodes

- A *state* is a (representation of) a physical configuration.
- A *node* is a data structure constituting part of a search tree
- A node includes: *parent*, *children*, *depth*, *path cost* $g(x)$.
- States do not have parents, children, depth, or path cost!
- The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.



Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!



Graph search algorithms

Basic idea:
similar to tree-search
keep a separate list of “explored” states

Graph search algorithms (cont'd)

```
function Graph-Search (problem)
returns a solution, or failure

    initialize the frontier using the initial state of problem
→   initialize the explored set to be empty
    loop do
        if the frontier is empty then return failure
        choose a leaf node and remove it from the frontier
        if the node contains a goal state
            then return the corresponding solution
→       add the node to the explored set
        expand the chosen node and add the resulting nodes to the frontier
→           only if not in the frontier or explored set
    end
```

Note: A → shows the lines that are added to the tree search algorithm.

Evaluating search strategies

- A strategy is defined by picking the *order of node expansion*
- Strategies are evaluated along the following dimensions:
  - *completeness*—does it always find a solution if one exists?
  - *time complexity*—number of nodes generated/expanded
  - *space complexity*—maximum number of nodes in memory
  - *optimality*—does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - b — maximum branching factor of the search tree
  - d — depth of the least-cost solution
  - m — maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search
- Bidirectional search

Breadth-first search

- Expand the shallowest unexpanded node
- Implementation: *frontier* is a FIFO queue, i.e., new successors go at end

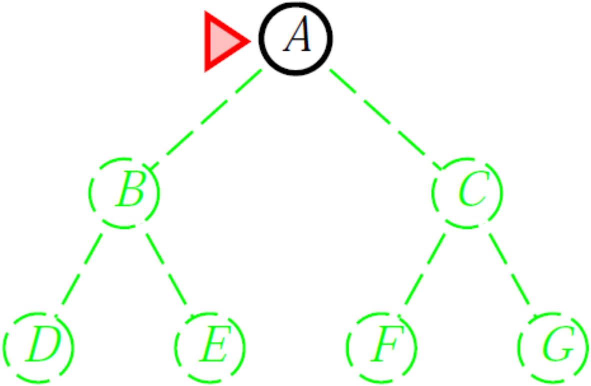
Progress of breadth-first search

Breadth-first search on a simple binary tree.

At each stage, the node to be expanded next is indicated by a marker.

The nodes that are already explored are gray.

The nodes with dashed lines are not generated yet.



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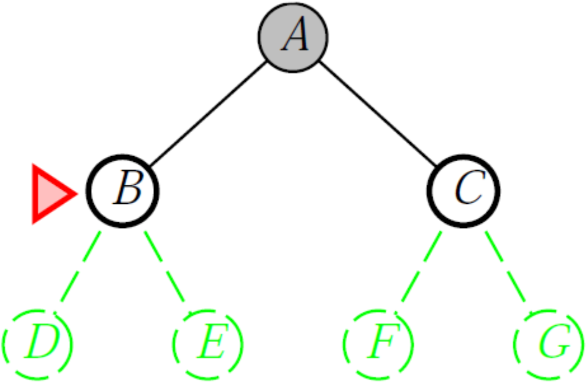

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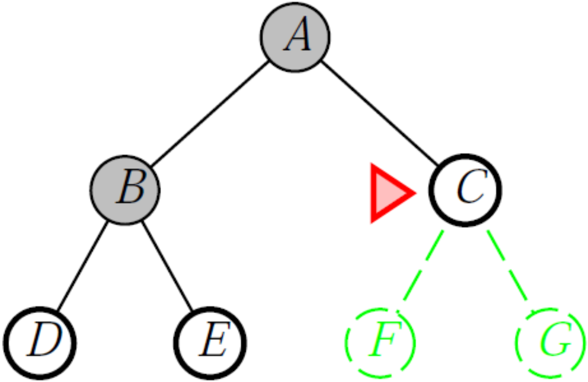

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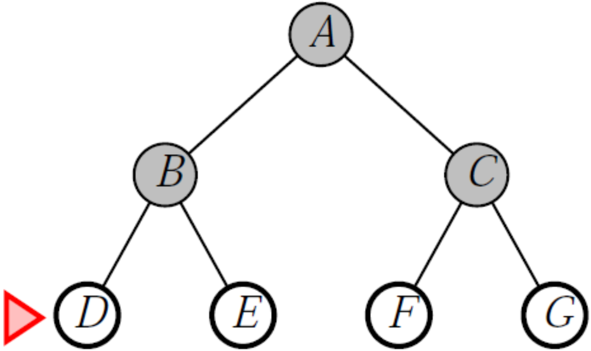


Properties of breadth-first search

- *Complete:* Yes (if b is finite)
- *Time:* $b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., number of nodes generated is exponential in d
- *Space:* $O(b^{d+1})$ (keeps every node in memory)
- *Optimal:* Yes (if cost $= 1$ per step)

Space is the big problem; can easily generate nodes at 100MB/sec so 24hrs $=$ 8604GB.

Breadth-first search algorithm

```
function Breadth-First-Search (problem)
returns a solution, or failure
    node ← a node with State=problem.Initial-State,
        Path-Cost = 0
    if problem.Goal-Test(node.State) then return Solution(node)
    frontier ← a FIFO queue with node as the only element
    explored ← an empty set
    loop do
        if Empty?(frontier) then return failure
        node ← pop(frontier) /* chooses the shallowest node in frontier */
        add node.State to explored
        for each action in problem.Actions(node.State) do
            child ← Child-Node (problem,node, action)
            if child.State is not in explored or frontier then
                if problem.Goal-Test (child.State) then
                    return Solution(child)
                frontier ← Insert (child, frontier)
```

Uniform-cost search

- Expand the least-cost unexpanded node
- Implementation: *frontier* is a queue ordered by path cost
- Equivalent to breadth-first if step costs are all equal

Properties of uniform-cost search

- *Complete:* Yes, if step cost $\geq \epsilon$
- *Time:* $\#$ of nodes with $g \leq$ cost of optimal solution, $O(b^{1+\lfloor C^*/\epsilon \rfloor})$
  where C^* is the cost of the optimal solution
- *Space:* $\#$ of nodes with $g \leq$ cost of optimal solution, $O(b^{1+\lfloor C^*/\epsilon \rfloor})$
- *Optimal:* Yes—nodes expanded in increasing order of $g(n)$

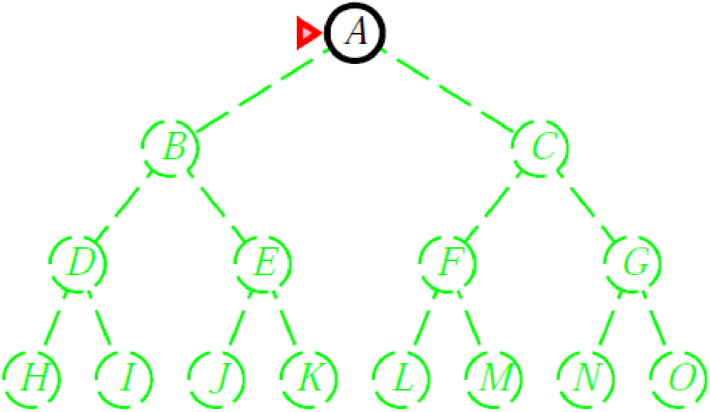
Uniform-cost search algorithm

```
function Uniform-Cost-Search (problem)
returns a solution, or failure
    node ← a node with State=problem.Initial-State,
        Path-Cost = 0
    if problem.Goal-Test(node.State) then return Solution(node)
    frontier ← a priority ordered by Path-Cost, with node as the only element
    explored ← an empty set
    loop do
        if Empty?(frontier) then return failure
        node ← pop(frontier) /* chooses the lowest-cost node in frontier */
        add node.State to explored
        for each action in problem.Actions(node.State) do
            child ← Child-Node (problem,node, action)
            if child.State is not in explored or frontier then
                frontier ← Insert (child, frontier)
            else if child.State is in frontier with higher Path-Cost then
                replace that frontier node with child
```

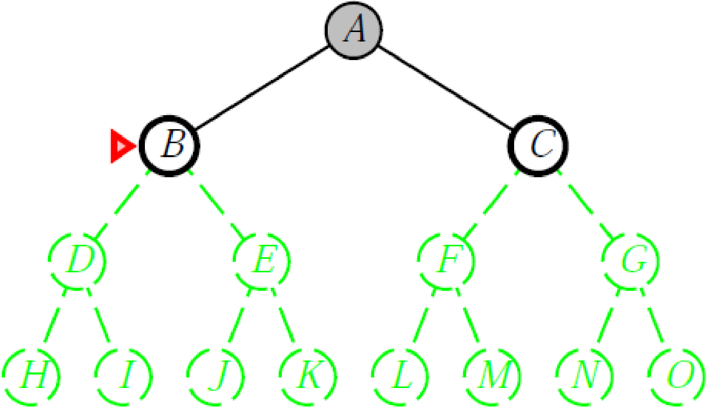
Depth-first search

- Expand deepest unexpanded node
- Implementation: *frontier* is a LIFO queue, i.e., put successors at front

Progress of depth-first search



Progress of depth-first search



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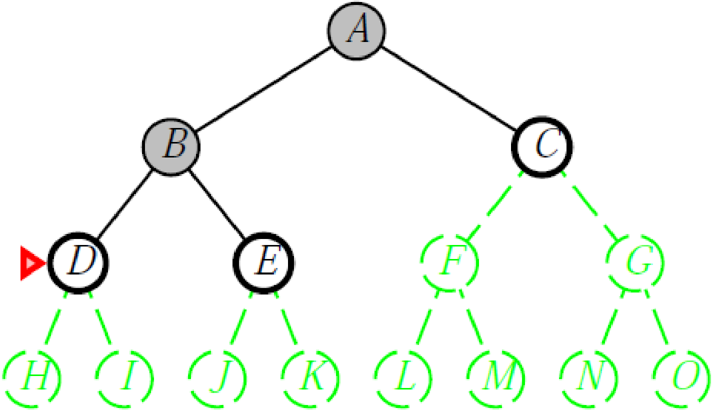

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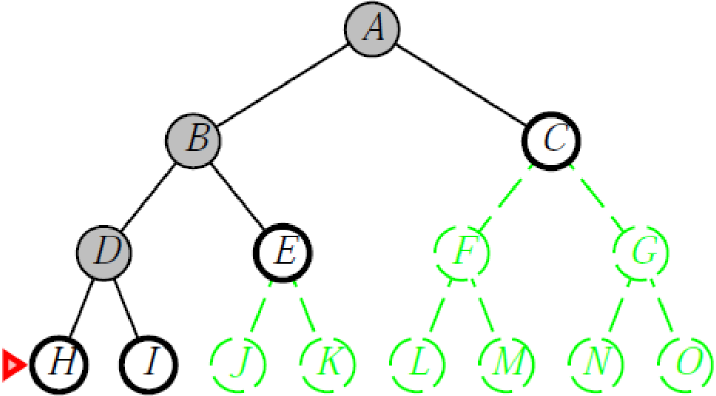

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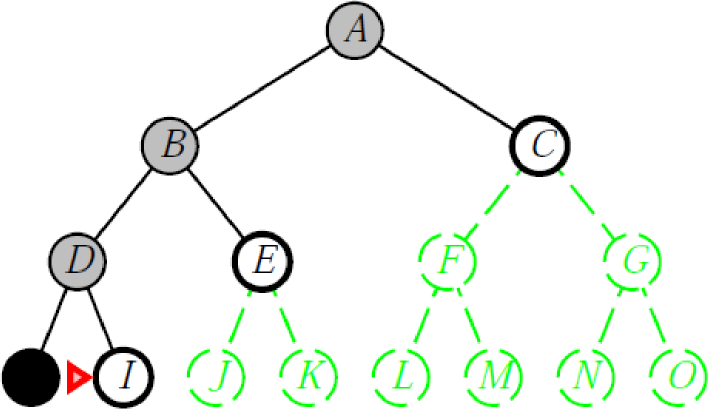

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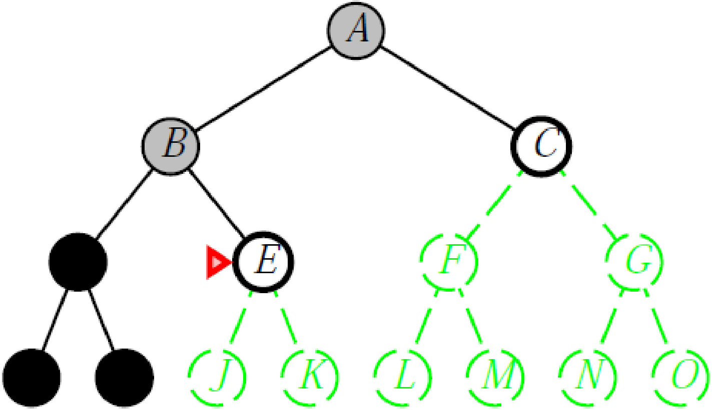

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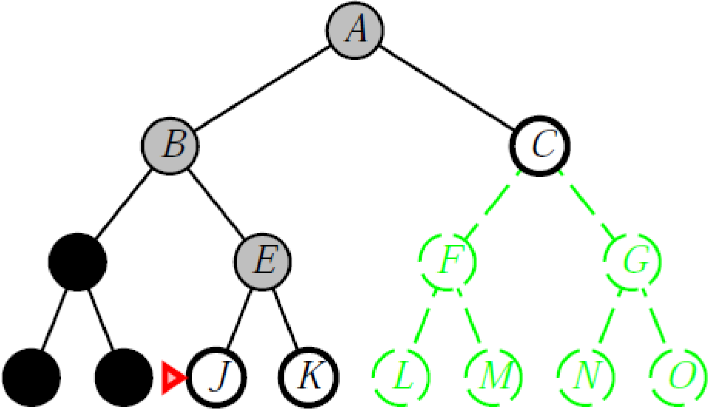

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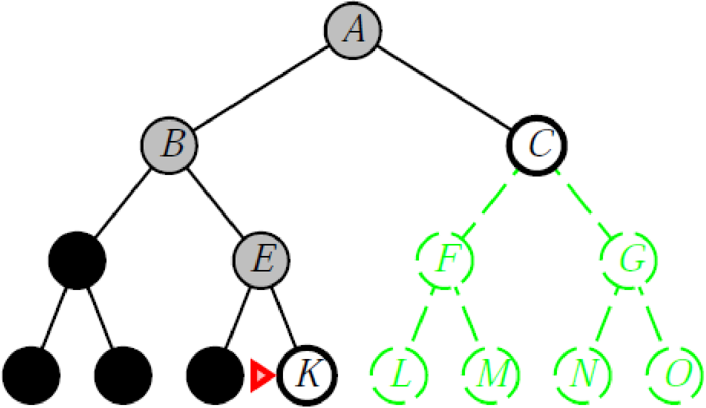

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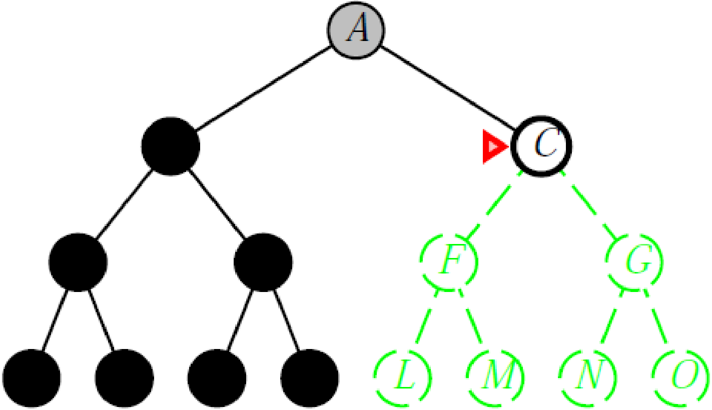

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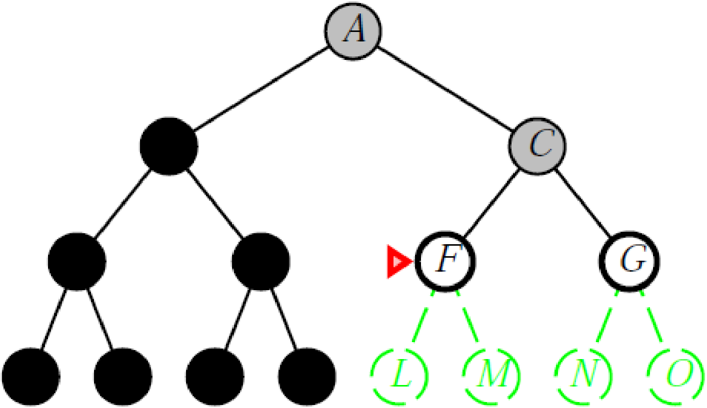

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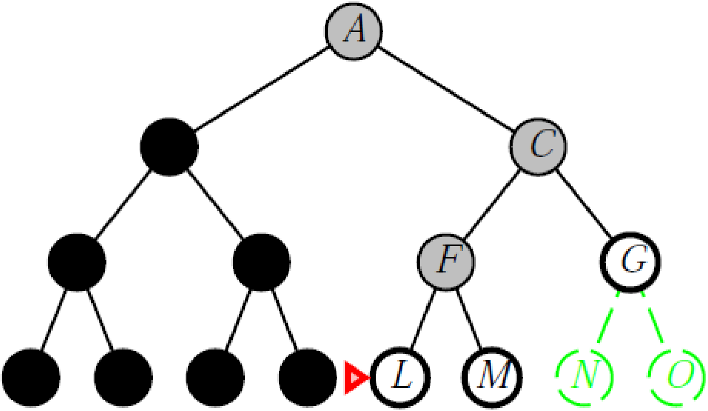

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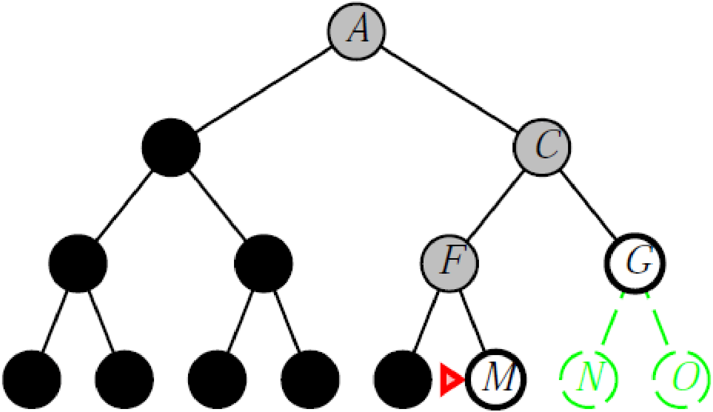


Properties of depth-first search

- *Complete:* No: fails in infinite-depth spaces, spaces with loops
  Modify to avoid repeated states along path
  $\Rightarrow$ complete in finite spaces
- *Time:* $O(b^m)$: terrible if m is much larger than d
  but if solutions are dense, may be much faster than breadth-first
- *Space:* $O(bm)$, i.e., linear space!
- *Optimal:* No

Depth-limited search

- It is equivalent to depth-first search with depth limit l, i.e., nodes at depth l have no successors
- implementation: a recursive implementation is shown on the next page

Properties of depth-limited search

- *Complete:* No (similar to DFS)
- *Time:* $O(b^l)$, where l is the depth-limit
- *Space:* $O(bl)$, i.e., linear space (similar to DFS)
- *Optimal:* No

Depth-limited search

```
function DEPTH-LIMITED-SEARCH (problem, limit)
returns a solution, or failure/cutoff
return RECURSIVE-DLS(MAKE-NODE( problem.INITIAL-STATE),
        problem, limit)

function RECURSIVE-DLS (node, problem, limit)
returns a solution, or failure/cutoff
    if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    else if limit $=$ 0 then return cutoff
    else
        cutoff-occurred? $\leftarrow$ false
        for each action in problem.ACTIONS(node.STATE) do
            child $\leftarrow$ CHILD-NODE (problem,node, action)
            result $\leftarrow$ RECURSIVE-DLS (child, problem,limit-1)
            if result $=$ cutoff then cutoff-occurred? $\leftarrow$ true
            else if result $\neq$ failure then return result
        if cutoff-occurred? then return cutoff else return failure
```

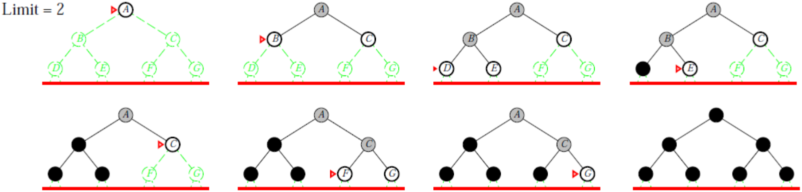
Iterative deepening search

- Do iterations of depth-limited search starting with a limit of 0. If you fail to find a goal with a particular depth limit, increment it and continue with the iterations.
- Terminate when a solution is found or if the depth-limited search returns *failure*, meaning that no solution exists.
- Combines the linear space complexity of DFS with the completeness property of BFS.

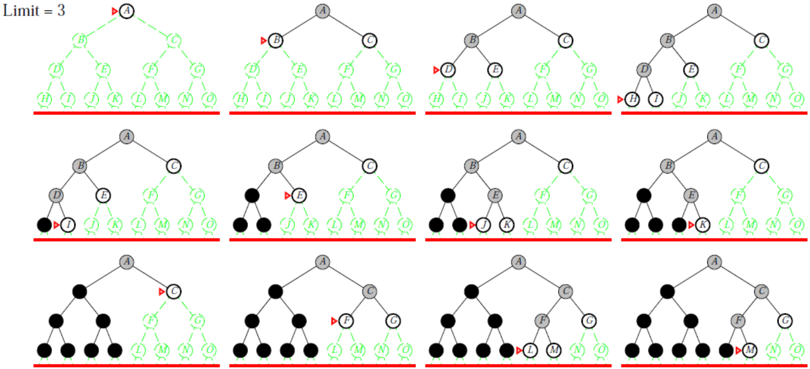
Iterative deepening search ($l = 0$)

Iterative deepening search ($l = 1$)

Iterative deepening search ($l = 2$)



Iterative deepening search ($l = 3$)



Properties of iterative deepening search

- *Complete:* Yes
- *Time:* $db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- *Space:* $O(bd)$
- *Optimal:* Yes, if step cost $= 1$
  Can be modified to explore uniform-cost tree

Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH(problem)
returns a solution, or failure
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH (problem, depth)
        if result ≠ cutoff then return result
```

Compare IDS and BFS

Numerical comparison of the number of nodes generated for $b = 10$ and $d = 5$, solution at the far right leaf:

$$\begin{aligned} N(\text{IDS}) &= 50 + 400 + 3,000 + 20,000 + 100,000 \\ &= 123,450 \\ N(\text{BFS}) &= 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 \\ &= 1,111,100 \end{aligned}$$

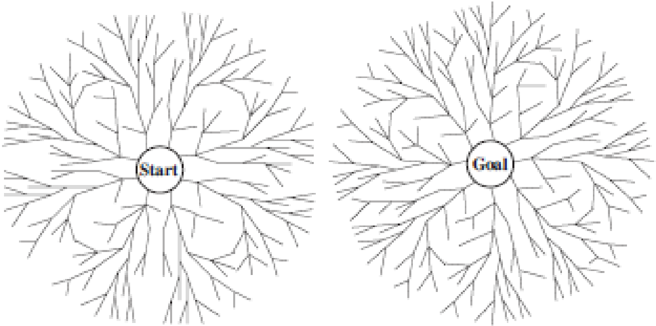
IDS does better because other nodes at depth d are not expanded. BFS can be modified to apply the goal test when a node is generated (rather than expanded).

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iter. Deep.
Complete?	Yes	Yes	No	Yes	Yes
Time	$O(b^{d+1})$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes*	Yes*	No	No	Yes

Bidirectional search

- Run two simultaneous states:
  one forward from the initial state
  one backward from the goal state
- Motivation: $b^{(\frac{d}{2})} + b^{\frac{d}{2}}$ is much less than b^d
- Implementation: Replace the goal check with a check to see whether the frontiers of the searches intersect



Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored.
- There are a variety of uninformed search strategies available.
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms.

Sources for the slides

- AIMA textbook (3^{rd} edition)
- AIMA slides (http://aima.cs.berkeley.edu/)