CS 4811 Artificial Intelligence Homework 4 — Logic

Due: Monday, March 19, 2012, beginning of class

Reminder: This is an individual assignment. All the work should be the author's and in accordance with the university's academic integrity policies. You are allowed to use any previously written source in preparing your answers, but if you use any other source than the textbook and the class notes, you should **credit the source** and write it on your assignment.

Question 1 *(10 points–2 points each)*.

We noted in class that the daily usage of words such as "and", "or", and "if ... then" might not always directly translate to logic. For each sentence below, give both a translation into propositional logic that preserves the intended meaning in English, and a straightforward translation as if the logical connectives had their regular logic meaning. Show an unintuitive consequence of the latter translation.

> **Part a.** They may or may not be able to find the missing boaters.
>
> **Part b.** Either the Dow goes up or there is more crisis coming.
>
> **Part c.** Siskel and Ebert are movie reviewers.
>
> **Part d.** The popcorn stand is open if you'd like to get some.
>
> **Part e.** You touch another painting and they'll kick us out of here.

Question 2 *(10 points)*.

We have defined 4 binary logical connectives in propositional logic: $\wedge, \vee, \Rightarrow$, and $\Leftrightarrow$. Are there any others that might be useful? How many binary connectives can there be? Explain your answers.

Question 3 *(10 points)*.

Suppose that the agent has progressed to the point shown in Figure 7.4(a), having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit and at most one can contain a wumpus. Following the example of Figure 7.5, construct the set of possible worlds. (You should find 32 of them). Mark the worlds in which the KB is true and those in which each of the following sentence is true:

$\alpha_2 =$ "There is no pit in [2,2]." $\alpha_3 =$ "There is a wumpus in [1,3]."

Question 4 *(10 points–2.5 points each)*. Prove each of the following assertions.

> **Part a.** α is valid if and only if $True \models \alpha$.
>
> **Part b.** For any α, $False \models \alpha$.
>
> **Part c.** $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.
>
> **Part d.** $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable.

Question 5 *(20 points–4 points each).*

Consider the following statements:
1. If there is an economic downturn, there will be fewer jobs.
2. If there are fewer jobs and John Doe has a good resume, he will get a good job.
3. If there are plenty of jobs ($\neg$ fewer jobs), John Doe will get a good job.
4. John Doe has a good resume.
5. There is an economic downturn.

> **Part a.** Convert the above statements into propositional logic by assigning a propositional literal to each.
>
> **Part b.** Use forward chaining to prove that John Doe will get a good job. Show the *and-or graph*.
>
> **Part c.** Show that if the knowledge base did not contain the last statement (there is an economic downturn), it would not be possible to use forward chaining to prove that John Doe will get a good job.
>
> **Part d.** Convert the first four statements into conjunctive normal form (CNF). The steps are the following:
>
> 1. Eliminate $\Leftrightarrow$s by replacing them with implications
> 2. Eliminate $\Rightarrow$ (implication)
> 3. Reduce the scope of negation using De Morgan's rules and double-negation
> 4. Convert expressions into conjunct of disjuncts form
> 5. Make each conjunct a separate clause
>
> **Part e.** Use resolution refutation on the CNF statements you created in part (d) to prove that John Doe will get a good job.

Question 6 *(5 points).*

Represent the following sentences in first-order logic using quantifiers. Remember to define a consistent vocabulary and write its semantics in English.

Part a. Some AI topics are symbolic.

Part b. Only one CS class is named "Artificial Intelligence."

Part c. Everybody who takes CS4811 needs to take three exams.

Part d. Every undergraduate student who is taking CS4811 must have taken CS3311.

Part e. Every student who is taking CS4811 knows somebody who likes AI.

Question 7 *(10 points).*

Consider a simplified representation of campus maps in first order logic. Assume that there are two interpretations. The first one (I_{mtu}) represents a simplified map of Michigan Tech (MTU), and the second one (I_{pitt}) represents a simplified map of the University of Pittsburgh.

In both maps, the cardinal directions are placed in the standard way. For instance, north is towards the top, and east is towards the right. Every name except "Main campus walkway", "Forbes Avenue", and "Schenley plaza" refer to buildings. "Main campus walkway" is a walkway, it can't be driven on. "Forbes Avenue" is a road that cars can drive on. "Schenley plaza" is a park.

ME–EM	Chemistry	EE	Dow	Dillman
Main campus walkway				
MUB	Library	Rekhi	Fisher	

MTU map

Brackenridge	William Pitt Union	Cathedral of Learning
Forbes Avenue		
Lawrence	Hillman Library	Schenley Plaza

Pitt map

For each sentence below, determine if it is true in interpretation I_{mtu} and in interpretation I_{pitt}. An *interpretation* specifies exactly which objects, relations, and functions are referred to by the constant, predicate, and function symbols (see page 292 for the definitions related to interpretations).

Part a. $\exists X$ is-building (X), $\exists X$ is-park (X)

Part b. $\forall X$ is-building$(X) \rightarrow$ taller-than(ME-EM, X)

Part c. $\exists X \forall Y$ is-park$(X) \wedge$ west-of(Y, X)

Part d. $\forall X, Y, Z$ west-of$(X, Y) \wedge$ west-of$(Y, Z) \rightarrow$ west-of(X, Z)

Question 8 *(5 points).*

State whether or not the following pairs of expressions are unifiable. If unifiable, show the mgu. If not, explain why. Show a non-mgu for one of the unifiable pairs.

Part a. in(X,Y) and in(Z, office-of(Z))
Part b. in(X,X) and in(Z, office-of(Z))
Part c. p(X,b,b) and p(a,Y,Z)
Part d. p(Y,Y,b) and p(Z,X,Z)
Part e. p(f(X,X),a) and p(f(Y,f(Y,a)),a)

Question 9 *(20 points).*

Consider the following sentences:

1. Whoever can read is literate.
2. Dolphins are not literate.
3. Some dolphins are intelligent.
4. Some who are intelligent cannot read.

Part a. Represent the above four statements in predicate logic using
R (X) for "X can read"
L (X) for "X is literate"
I (X) for "X is intelligent"
D (X) for "X is a dolphin"

Part b. Set up sentences so that the fourth can be proven using the first three employing resolution refutation. Then convert the sentences to clause form using the following steps:

1. Eliminate $\rightarrow$ (implication)
2. Reduce the scope of negation
3. Standardize variables apart
4. Move all quantifiers to the left without changing their order
5. Eliminate existential quantifiers (Skolemize)
6. Drop all universal quantifiers
7. Convert expressions into conjunct of disjuncts form
8. Make each conjunct a separate clause
9. Standardize the variables apart again

Part c. Prove the fourth statement using resolution refutation.