# CS 4811 Artificial Intelligence
# Homework 4 — First-Order Logic

**Due: Tuesday, March 24, 2009, beginning of class**

**Reminder:** This is an individual assignment. All the work should be the author's and in accordance with the university's academic integrity policies. You are allowed to use any previously written source in preparing your answers, but if you use any other source than the textbook and the class notes, you should specify it on your assignment.

**Question 1** *(15 points).*

Represent the following sentences in first-order logic using quantifiers. Remember to define a consistent vocabulary and write its semantics in English.

**(a)** Some AI topics are symbolic.

**(b)** Only one CS class is named "Artificial Intelligence."

**(c)** Everybody who takes CS4811 needs to take three exams.

**Question 2** *(30 points).*

Consider a simplified representation of campus maps in first order logic. Assume that there are two interpretations. The first one ($I_{mtu}$) represents a simplified map of Michigan Tech (MTU), and the second one ($I_{pitt}$) represents a simplified map of the University of Pittsburgh.

In both maps, the cardinal directions are placed in the standard way. For instance, north is towards the top, and east is towards the right. Every name except "Main campus walkway", "Forbes Avenue", and "Schenley plaza" refer to buildings. "Main campus walkway" is a walkway, it can't be driven on. "Forbes Avenue" is a road that cars can drive on. "Schenley plaza" is a park.

| | | | | |
|---|---|---|---|---|
| ME–EM | Chemistry | EE | Dow | Dillman |
| Main campus walkway | | | | |
| MUB | Library | Rekhi | Fisher | |

MTU map

| | | |
|---|---|---|
| Brackenridge | William Pitt Union | Cathedral of Learning |
| Forbes Avenue | | |
| Lawrence | Hillman Library | Schenley Plaza |

Pitt map

For each sentence below, determine if it is true in interpretation $I_{mtu}$ and in interpretation $I_{pitt}$ (see page 246 for the definitions related to *interpretations*).

**Part a.** $\exists X$ is-building $(X)$

**Part b.** $\exists X$ is-park $(X)$

**Part c.** $\forall X$ north-of(X, Schenley-Plaza) $\vee$ west-of(X, Schenley-Plaza)

**Part d.** $\forall X$ is-building$(X) \rightarrow$ taller-than(ME-EM, $X$)

**Part e.** $\exists X \forall Y$ is-park$(X) \wedge$ west-of$(Y, X)$

**Part f.** $\forall X, Y, Z$ west-of$(X, Y) \wedge$ west-of$(Y, Z) \rightarrow$ west-of$(X, Z)$

**Question 3** *(30 points).*

State whether or not the following pairs of expressions are unifiable. If unifiable, show the mgu. If not, explain why. Show a non-mgu for one of the unifiable pairs.

**Part a.** in(X,Y) and in(Z, office-of(Z))
**Part b.** in(X,X) and in(Z, office-of(Z))
**Part c.** in(X,Y) and in(Z, office-of(W))
**Part d.** p(X,b,b) and p(a,Y,Z)
**Part e.** p(Y,Y,b) and p(Z,X,Z)
**Part f.** p(f(X,X),a) and p(f(Y,f(Y,a)),a)

**Question 4** *(25 points).*

Consider the following sentences:

1. Whoever can read is literate.
2. Dolphins are not literate.
3. Some dolphins are intelligent.
4. Some who are intelligent cannot read.

**Part a.** Represent the above four statements in predicate logic using
`R (X)` for "X can read"
`L (X)` for "X is literate"
`I (X)` for "X is intelligent"
`D (X)` for "X is a dolphin"

**Part b.** Set up sentences so that the fourth can be proven using the first three employing resolution refutation. Then convert the sentences to clause form using the following steps:

1. Eliminate $\rightarrow$ (implication)
2. Reduce the scope of negation
3. Standardize variables apart
4. Move all quantifiers to the left without changing their order
5. Eliminate existential quantifiers (Skolemize)
6. Drop all universal quantifiers
7. Convert expressions into conjunct of disjuncts form
8. Make each conjunct a separate clause
9. Standardize the variables apart again

**Part c.** Prove the fourth statement using resolution refutation.