Question 1 (15 points—3 points each).

Consider each sentence below. State whether it is a legal sentence (wff) in propositional logic. State whether it is a legal sentence (wff) first-order logic (FOL). Explain your answer. Assume that P and Q represent propositions or predicates, and X and Y represent variables.

(a) (P) → (Q)
(b) P → Q
(c) P → X
(d) P (Q)
(e) P (Y)

Question 2 (12 points—3 points each).

We noted in class that the daily usage of words such as ”and”, ”or”, and ”if ... then” might not always directly translate to logic. For each sentence below, give both a translation into logic (propositional or first-order) that preserves the intended meaning in English, and a straightforward translation (as if the logical connectives had their regular first-order logic meaning). Show an unintuitive consequence of the latter translation.

(a) I may or may not be able to visit the snow statues.
(b) The snow statues are still up if you’d like to go.
(c) I’ll go with you if you go.
(d) You touch another statue and we are leaving.

Question 3 (9 points—3 points each).

For each of the following statements decide whether it is valid, satisfiable, or unsatisfiable.

(a) cold → hot-cocoa
(b) cold ∨ hot-cocoa ∨ ¬hot-cocoa
(c) (snow → ski) → (¬snow → ¬ski)
Question 4 (10 points).

We have defined 4 binary logical connectives in propositional logic: ∧, ∨, →, and ↔. Are there any others that might be useful? How many binary connectives can there be? Explain your answers.

Question 5 (9 points–3 points each).

Represent the following sentences in first-order logic using quantifiers. Remember to define a consistent vocabulary and write its semantics in English.

(a) Some statues have a lot of detail.

(b) Only one statue has a ship.

(c) Everybody who takes CS4811 needs to take three exams.

Question 6 (18 points–3 points each).

Consider a representation of campus maps in first order logic. Assume that there are two interpretations. The first one ($I_{mtu}$) represents a simplified map of MTU and is shown below.

<table>
<thead>
<tr>
<th>ME–EM</th>
<th>Chemistry</th>
<th>EE</th>
<th>Dow</th>
<th>Dillman</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main campus walkway</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MUB</td>
<td>Library</td>
<td>Rekhi</td>
<td>Fisher</td>
<td></td>
</tr>
</tbody>
</table>

MTU map

The second one ($I_{pitt}$) represents a simplified map of Pitt and is shown below.
In both maps, the cardinal directions are placed in the standard way. For instance, north is towards the top, and east is towards the right. Every name except “Main campus walkway”, “Forbes Avenue”, and “Schenley plaza” refer to buildings. “Main campus walkway” is a walkway, it can’t be driven on. “Forbes Avenue” is a road that cars can drive on. “Schenley plaza” is a park.

For each sentence below, determine if it is true in interpretation I_{mtu} and in interpretation I_{pitt}.

(a) \exists X \text{ is-building}(X)
(b) \exists X \text{ is-park}(X)
(c) \forall X \text{ north-of}(X, \text{ Schenley-Plaza}) \lor \text{ west-of}(X, \text{ Schenley-Plaza})
(d) \forall X \text{ is-building}(X) \rightarrow \text{ taller-than}(\text{ME-EM, } X)
(e) \exists X \forall Y \text{ is-park}(X) \land \text{ west-of}(Y, X)
(f) \forall X, Y, Z \text{ west-of}(X, Y) \land \text{ west-of}(Y, Z) \rightarrow \text{ west-of}(X, Z)

**Question 7 (12 points–3 points each).**

Say whether or not the following pairs of expressions are unifiable. If unifiable, show the mgu and a non-mgu, if not, explain why. Note that a non-mgu might not always exist.

(a) p(X,b,b) and p(a,Y,Z)
(b) p(Y,Y,b) and p(Z,X,Z)
(c) p(f(X,X),a) and p(f(Y,f(Y,a)),a)
(d) q(X) and \neg q(a)
**Question 8** *(15 points).*

Consider the following sentences:

1. Whoever can read is literate.
2. Dolphins are not literate.
3. Some dolphins are intelligent.
4. Some who are intelligent cannot read.

(a) Represent the above four statements in predicate logic using

\[ R(X) \] for "X can read"
\[ L(X) \] for "X is literate"
\[ I(X) \] for "X is intelligent"
\[ D(X) \] for "X is a dolphin"

(b) Set up sentences so that the fourth can be proven using the first three employing resolution refutation. Then convert the sentences to clause form using the following steps:

1. Eliminate \( \rightarrow \) (implication)
2. Reduce the scope of negation
3. Standardize variables apart
4. Move all quantifiers to the left without changing their order
5. Eliminate existential quantifiers (Skolemize)
6. Drop all universal quantifiers
7. Convert expressions into conjunct of disjuncts form
8. Make each conjunct a separate clause
9. Standardize the variables apart again

(c) Prove the fourth statement using resolution.

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