1. (40 points) Construct PDAs that accept each of the following languages.

Explain how the PDA works: write the algorithm it follows, label the specific portions of the machine with the task performed (5 points for each machine).

(a) \( \{a^i b^j \mid 0 \leq i \leq j\} \)

(b) \( \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + k = j\} \)

2. (20 points) Let \( M \) be the TM in Example 8.2.2 on page 261 (the machine for \( a^i b^j c^k \)). Show the computation sequence for the strings \( abc \) and \( aabc \).
3. (20 points) Construct a TM that takes an input consisting of a sequence of $a$’s followed by fewer or equal number of $b$’s; and outputs a string where the number of $b$’s is the same as the original number of $a$’s.

The input format is: $\{a^ib^j | i, j \geq 0 \text{ and } i \geq j\}$

The output format is: $\{a^ib^i | i \geq 0\}$

For example:
If the input is ‘BaaaaabbB’, the output should be ‘BaaaabbbB’.
If the input is ‘BaaabbbB’, the output should stay the same: ‘BaaabbbB’.

You may assume that the input will be in the desired format. There is no need to check for errors.

Write the high-level algorithm executed by the machine and label the sections (5 points).

4. (20 points) Construct a TM that accepts the following language.
Write the high-level algorithm executed by the machine and label the sections (5 points).

$\{a^ib^j c^k | i + j = k\}$