

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

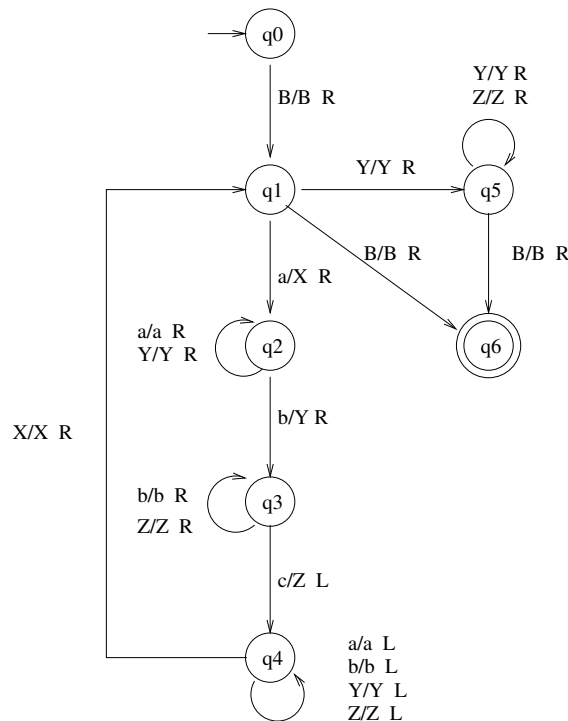
1. (40 points) Construct PDAs that accept each of the following languages.

Explain how the PDA works: write the algorithm it follows, label the specific portions of the machine with the task performed (5 points for each machine).

(a) $\{a^i b^j \mid 0 \leq i \leq j\}$

(b) $\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i + k = j\}$

2. (20 points) Let M be the TM in Example 8.2.2 on page 261 (the machine for $a^i b^i c^i$). Show the computation sequence for the strings abc and $aabc$.



3. (20 points) Construct a TM that takes an input consisting of a sequence of a 's followed by fewer or equal number of b 's; and outputs a string where the number of b 's is the same as the original number of a 's.

The input format is: $\{a^i b^j \mid i, j \geq 0 \text{ and } i \geq j\}$

The output format is: $\{a^i b^i \mid i \geq 0\}$

For example:

If the input is ' $BaaaaabbB$ ', the output should be ' $BaaaaabbbbB$ '.

If the input is ' $BaaabbbbB$ ', the output should stay the same: ' $BaaabbbbB$ '.

You may assume that the input will be in the desired format. There is no need to check for errors.

Write the high-level algorithm executed by the machine and label the sections (**5 points**).

4. (20 points) Construct a TM that accepts the following language.

Write the high-level algorithm executed by the machine and label the sections (**5 points**).

$\{a^i b^j c^k \mid i + j = k\}$