CS3311 Homework 9

Due: Wednesday, March 22, 2017, by class time, 12:05pm
Submission: Typed, pdf on Canvas
(scanned submissions are not allowed)

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(60 points)* Give a context-free grammar for each of the following languages *(8 points each)*. Explain how the grammar works *(2 points each)*.

(a) $L = \emptyset$

(b) $L = \{\lambda\}$

(c) $L = \{w \mid w \in \{a, b, c\}^*, w \text{ does not contain substring } bc\} = c^*(b \cup ac^*)^*$

(d) ROLL is the language generated by the following recursive definition. It gives a few basic drum roll patterns. R stands for "right" and L stands for "left."

> **basis:** "single-stroke:" $\in$ ROLL, "double-stroke:" $\in$ ROLL,
> "silly-stroke:" $\in$ ROLL.
>
> **recursive step**: If ($w \in$ ROLL and w contains "single") then wRL is in ROLL
> If ($w \in$ ROLL and w contains "double") then $wRRLL$ is in ROLL
> If ($w \in$ ROLL and $w = xy$ where $x =$ "silly-stroke:") then $xRyL$ is in ROLL
>
> **closure**: A string $w \in$ ROLL only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(e) $L = \{a^n b^m c^{2n+m} \mid n, m \geq 0\}$

(f) $L = \{a^n b^m \mid n \neq m, n \geq 0, m \geq 0\}$
(Hint: "not equal to" means "less than or greater than".)

2. *(5 + 5 points)* Consider the following grammar:

$$S \to aSA \mid \lambda$$
$$A \to bA \mid \lambda$$

(a) Prove the grammar is ambiguous by finding a string that has two distinct leftmost derivations. Show the two derivations.

(b) Build the derivation trees for the derivations in Part (a).

3. *(5 + 5 + 10 points)* Consider the following grammar G over $\Sigma = \{a, b\}$.

$$S \to AB \,|\, aB$$
$$A \to aB \,|\, BS$$
$$B \to bB \,|\, b \,|\, \lambda$$

(a) Transform the grammar into $G1$ so that the recursion to the start symbol is removed.

(b) Show the set of nullable variables in the **new grammar** $G1$.

(c) Construct an essentially noncontracting grammar G_L (with a non-recursive start symbol) equivalent to $G1$. An "essentially noncontracting grammar" is a grammar that has no λ-rules.

4. *(5+5 points)* Consider the following grammar G. Note that the grammar does not contain λ-rules except at S.

$$S \to aSb \,|\, DEF \,|\, D \,|\, \lambda$$
$$D \to E \,|\, EF \,|\, abEF$$
$$E \to eEff \,|\, a \,|\, F$$
$$F \to ffFe \,|\, a$$

(a) Use algorithm 4.3.1 to construct the `CHAIN` sets for the variables in V.

(b) Construct an equivalent grammar G_c that does not contain chain rules.