1. (2+8 points) State whether the following equality is true or false. Explain your answer.

\[(a \cup b)^* = (a^*) \cup (b^*)\]

2. (2+8 points) State whether the following equality is true or false. Explain your answer.

\[(a \cup b)^* = (a \cup (a*b))^*\]

3. (60 points) Give a regular expression for the following languages.

(a) The set of strings over \{1, 2, 3, a, b, c\} that contain exactly two numbers and the sum of the numbers is even.

(c) The set of strings over \{a, b, c\} in which all the a’s precede the b’s, which in turn precede the c’s. It is possible that there are no a’s, or b’s, or c’s and the string is empty.

(d) The set of strings over \{a, b, c\} in which all the a’s precede the b’s, which in turn precede the c’s. It is possible that there are no a’s, or b’s, or c’s, but \lambda is not in the language.

(e) The set of strings over \{1, 2, 3, a\} that do not begin with 123.

4. (20 points) The following DFA \(M_1\) accepts all strings that end in ‘ab’.

Give a 5-tuple that formally describes the DFA.
Write the transition function as a table.