CS3311 Homework 3

Due date: Wednesday, February 1, 2017, in class

The answers must be the original work of the author. While discussion and studying with others is permitted and encouraged, the final work should be done individually. You are not allowed to do homework in groups. You are allowed to build on the material supplied in class. Any other source must be specified clearly.

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1. *(5+5+5+10 points)* Let L be the language over $\Sigma = \{a, b, d, e\}$ generated by the following recursive definition:

> **basis:** $d \in L$, $e \in L$
>
> **recursive step**: If ($w \in L$ and w contains d) then aaw is in L and wab is in L.
> If ($w \in L$ and w contains e) then $aawab$ is in L.
>
> **closure**: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(a) Give the sets L_1, L_2, and L_3 generated by the recursive definition. Note that $L_0 = \{d, e\}$.

(b) For each of the following five strings, tell whether the string is in L and indicate the reason.

$\lambda, aad, aae, ada, aaeab, abeaa$

(c) Give an implicit definition of the set of strings defined by the recursive definition. An implicit definition describes the pattern of the strings in a set by using a vertical bar to denote "such that". For example: $\{x | x \in \Sigma^*$, $x = a^i b^i$, and i is even $\}$.

(d) Use induction to prove that all the strings in L have an odd length.

You must present the proof based on the recursive definition and on the number of applications of the recursive step. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.

2. *(30 points)* Let L over $\Sigma = \{1, 2, 3, a, b, c, -\}$ be the language of names where every name has to begin with a letter (a, b, or c).

(a) Give a recursive definition for L.

(b) Give a regular set for L.

3. *(45 points)* Give a regular set for the following languages.

(a) The set of strings over $\{1, 2, a, b, c\}$ that start with "a" and end with "1". Strings can have a length of one or greater.

(b) The set of strings over $\{1, 2, a, b, c\}$ that start and end with a number. Strings can have a length of one or greater.

(c) The set of strings over $\{a, b, c\}$ in which all the a's precede the b's, which in turn precede the c's. It is possible that there are no a's, or b's, or c's or the string is empty.