1. (5+5+5+10 points) Let $L$ be the language over $\Sigma = \{a, b, d, e\}$ generated by the following recursive definition:

**basis:** $d \in L$, $e \in L$

**recursive step:** If ($w \in L$ and $w$ contains $d$) then $aaw$ is in $L$ and $wab$ is in $L$. If ($w \in L$ and $w$ contains $e$) then $aawab$ is in $L$.

**closure:** A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(a) Give the sets $L_1$, $L_2$, and $L_3$ generated by the recursive definition. Note that $L_0 = \{d, e\}$.

(b) For each of the following five strings, tell whether the string is in $L$ and indicate the reason.

$\lambda$, $aad$, $aae$, $ada$, $aaeab$, $abeaa$

(c) Give an implicit definition of the set of strings defined by the recursive definition. An implicit definition describes the pattern of the strings in a set by using a vertical bar to denote “such that”. For example: $\{x | x \in \Sigma^*, x = a^i b^i, and i is even \}$.

(d) Use induction to prove that all the strings in $L$ have an odd length.

You must present the proof based on the recursive definition and on the number of applications of the recursive step. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.

2. (30 points) Let $L$ over $\Sigma = \{1, 2, 3, a, b, c, -\}$ be the language of names where every name has to begin with a letter ($a, b, or c$).

(a) Give a recursive definition for $L$.

(b) Give a regular set for $L$.

3. (45 points) Give a regular set for the following languages.

(a) The set of strings over $\{1, 2, a, b, c\}$ that start with “$a$” and end with “$1$”. Strings can have a length of one or greater.

(b) The set of strings over $\{1, 2, a, b, c\}$ that start and end with a number. Strings can have a length of one or greater.

(c) The set of strings over $\{a, b, c\}$ in which all the $a$’s precede the $b$’s, which in turn precede the $c$’s. It is possible that there are no $a$’s, or $b$’s, or $c$’s or the string is empty.