CS3311 Homework 2 Due date: Wednesday, September 20, 2017, by class time, 1:05pm
Submission: Typed, on Canvas (scanned submissions are not allowed)

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

---

1. *(50 points)*

Consider the set S_1 constructed recursively.

> **(i) Basis:** $[1, 1] \in S_1$
>
> **(ii) Recursive step:** If $[n, m] \in S_1$, then $[n + 1, m + 2(n + 1) - 1] \in S_1$.
>
> **(iii) Closure:** S_1 consists of exactly the elements that can be obtained by starting with the basis elements of S_1 and applying the recursive step finitely many times to construct new elements of S_1.

Prove using induction that for every pair in S_1, the second member is the square of the first. In other words, for all $[n, m] \in S_1$ $m = n^2$.

2. *(50 points)* Consider the set S_2 constructed recursively. The first member of the basis sequence is a number, and the second member is a string. The recursive step performs an arithmetic addition on the first member and string concatenation on the second member. The symbols a, b are characters, not variables.

> **(i) Basis:** $[1, a] \in S_2$ and $[1, b] \in S_2$
>
> **(ii) Recursive step:** If $[n, w] \in S_2$, then $[n + 2, awa] \in S_2$ and $[n + 2, bwb] \in S_2$.
>
> **(iii) Closure:** S_2 consists of exactly the elements that can be obtained by starting with the basis elements of S_2 and applying the recursive step finitely many times to construct new elements of S_2.

Use induction to prove that the first member of each pair represents the length of the string which is the second member. In other words, for all $[n, m] \in S_2$ $n =$length(m).