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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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Consider the following sets for questions **1** and **2**:

$$X = \{a, 2, \{a\}, [a, a], [a, \emptyset]\} \quad Y = \{a, 3, [a], [\emptyset, a], \emptyset\}$$

**1.** (10 points) Write out each of the sets listed below.

- (a)  $X \cup Y$
- (b)  $X \cap Y$
- (c)  $X - Y$
- (d)  $P(X - \{[a, a], [a, \emptyset]\})$
- (e)  $\{a, 1\} \times \{a, [a, a], \emptyset\}$

**2.** (20 points) State whether the following propositions are TRUE or FALSE.

- (a)  $a \in X$
- (b)  $\{a\} \in X$
- (c)  $a \in Y$
- (d)  $\{a\} \in Y$
- (e)  $\emptyset \in X$
- (f)  $\emptyset \in Y$
- (g)  $\emptyset \subseteq Y$
- (h)  $\{\emptyset\} \subseteq X$
- (i)  $\{\emptyset\} \subseteq Y$
- (j)  $\{[a, a]\} \in X \times X$

*Please turn the page over for additional questions.*

**3. (20 points)**

(a) Write the first 5 elements of the set  $S_1$  defined recursively. Put the basis elements as the first members. Assume that the arithmetic computations defined in the recursive step will be performed to obtain the new elements of  $S_1$ .

(i) **Basis:**  $[1, 1] \in S_1$

(ii) **Recursive step:** If  $[n, m] \in S_1$ , then  $[n + 1, m + 2(n + 1) - 1] \in S_1$ .

(iii) **Closure:**  $S_1$  consists of exactly the elements that can be obtained by starting with the basis elements of  $S_1$  and applying the recursive step finitely many times to construct new elements of  $S_1$ .

(b) Write the first 6 elements of the set  $S_2$  defined recursively. Put the basis elements as the first members. The first member of the basis sequence is a number, and the second member is a string. The recursive step performs an arithmetic addition on the first member and string concatenation on the second member. The symbols  $a, b$  are characters, not variables.

(i) **Basis:**  $[1, a] \in S_2$  and  $[1, b] \in S_2$

(ii) **Recursive step:** If  $[n, w] \in S_2$ , then  $[n + 2, awa] \in S_2$  and  $[n + 2, bwb] \in S_2$ .

(iii) **Closure:**  $S_2$  consists of exactly the elements that can be obtained by starting with the basis elements of  $S_2$  and applying the recursive step finitely many times to construct new elements of  $S_2$ .

**4. (20 points)** Give a recursive definition for the following sets. You may use addition and other operators to generate new integer elements. There are no restrictions on the conditions checked via an `if` statement. Explain the “pattern” you are using.

**No points will be given to answers without an accompanying explanation.**

(a)  $\{2, 8, 20, 44, \dots\}$

(b)  $\{2, 8, 20, 44, \dots\} \cup \{1, 5, 9, 13, \dots\}$

*There are additional questions on the next page.*

5. (10 points) Consider the following infinite loop in pseudocode that fills out a set  $D$ . Note that  $\leftarrow$  is used as the assignment operator,  $k$  is a string, and “+” represents string concatenation.

```
 $D \leftarrow \emptyset$   
 $k \leftarrow$  "Ice cream is good, yum"  
while (true) do {  
     $D \leftarrow D \cup \{k\}$   
     $k \leftarrow k +$  "m"  
}
```

List the elements of set  $D$  after 0, 1, and 2 iterations of the loop.

6. (20 points)

Consider the following infinite loop in pseudocode that fills out a set  $C$ . Note that  $\leftarrow$  is used as the assignment operator,  $i$  is an integer,  $w$  is a string. “ $a$ ” and “ $b$ ” are characters (symbols) that are in the string.

“ $a$ ”+ $w$ + “ $b$ ” means concatenate letter  $a$  to the left of  $w$  and concatenate letter  $b$  to the right of  $w$ . “+” represents addition for numbers and concatenation for strings.

```
 $C \leftarrow \emptyset$   
 $i \leftarrow 2$   
 $w \leftarrow$  "ab"  
while (true) do {  
     $C \leftarrow C \cup \{i\} \cup \{w\}$   
     $i \leftarrow i + 3$   
     $w \leftarrow$  "a"+ $w$ + "b"  
}
```

(a) List the elements of set  $C$  after 0, 1, 2, and 3 iterations of the loop.

(b) Give a recursive definition for the set  $C$ . In the recursive definition, use  $awb$  rather than “ $a$ ”+ $w$ + “ $b$ ”.