CS3311 Homework 2 Due date: Wednesday, January 27, 2016, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(25 points)* Use induction to prove that $\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$

Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.

Hint: Two ways of making a value larger are to subtract a smaller value from the original and to add a larger value to the original.

2. *(25 points)* Consider the following Python program and use induction on the number of iterations of the `for` loop to prove that the program always prints
`True:  y is less than or equal to z.`

You must present the proof based on the code and on the number of iterations of the `for` loop. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**. The loop is an implementation of the above question.

```
import operator

n = input('Pick a number greater than 0: ')
print ('You entered %s. \n' % (n))

y=0.0
for i in range (1,n+1):   # executes for i = 1, 2, ..., n
  x = 1.0 / (i*i)
  y = y+x
  z = 2.0 - (1.0/i)

if operator.le (y,z):     # if y is less-than-or-equal z
  print "True: y is less than or equal to z."
else:
  print "False: y is greater than z."
```

Please turn the page over for additional questions.

3. *(5+20 points)* Let L be the language generated by the following recursive definition:

> **basis:** "It's cold: brrr" $\in L$
>
> **recursive step**: If $(w \in L)$ then $wrr \in L$
>
> **closure**: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(a) Give the first three elements of L.

(b) Use induction to prove that all the strings in L above have an odd length. To save you from counting, the length of the string in the basis is 15.

4. *(5+5+5+10 points)* Let L be the language over $\Sigma = \{a, b, d, e\}$ generated by the following recursive definition:

> **basis:** $d \in L$, $e \in L$
>
> **recursive step**: If ($w \in L$ and w contains d) then aaw is in L and wab is in L.
> If ($w \in L$ and w contains e) then $aawab$ is in L.
>
> **closure**: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(a) Give the sets L_1, L_2, and L_3 generated by the recursive definition. Note that $L_0 = \{d, e\}$.

(b) For each of the following five strings, tell whether the string is in L and indicate the reason.

$\lambda, aad, aae, ada, aaeab, abeaa$

(c) Give an implicit definition of the set of strings defined by the recursive definition. An implicit definition describes the pattern of the strings in a set by using a vertical bar to denote "such that". For example: $\{x | x \in \Sigma^*$, $x = a^i b^i$, and i is even $\}$.

(d) Use induction to prove that all the strings in L have an odd length.