
The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

1. (10 + 10 points) Consider the following grammar:

$$\begin{aligned} S &\rightarrow aSA \mid \lambda \\ A &\rightarrow bA \mid \lambda \end{aligned}$$

(a) Prove the grammar is ambiguous by finding a string that has two distinct leftmost derivations. Show the two derivations.

(b) Build the derivation trees for the derivations in Part (a).

2. (5 + 5 + 10 points) Consider the following grammar G over $\Sigma = \{a, b\}$.

$$\begin{aligned} S &\rightarrow AB \mid aB \\ A &\rightarrow aB \mid BS \\ B &\rightarrow bB \mid b \mid \lambda \end{aligned}$$

(a) Transform the grammar into G_1 so that the recursion to the start symbol is removed.

(b) Show the set of nullable variables in the **new grammar** G_1 .

(c) Construct an essentially noncontracting grammar G_L (with a non-recursive start symbol) equivalent to G_1 . An “essentially noncontracting grammar” is a grammar that has no λ -rules.

3. (5+10 points) Consider the following grammar G . Note that the grammar does not contain λ -rules except at S .

$$\begin{aligned} S &\rightarrow aSb \mid DEF \mid D \mid \lambda \\ D &\rightarrow E \mid EF \mid abEF \\ E &\rightarrow eE ff \mid a \mid F \\ F &\rightarrow ffFe \mid a \end{aligned}$$

(a) Use algorithm 4.3.1 to construct the CHAIN sets for the variables in V .

(b) Construct an equivalent grammar G_e that does not contain chain rules.

Please turn the page over for additional questions.

4. (5+10 points) Consider the following grammar G :

$$\begin{aligned} S &\rightarrow a \mid aA \mid BC \\ A &\rightarrow aB \mid b \\ B &\rightarrow Aa \\ C &\rightarrow cCD \\ D &\rightarrow ddd \end{aligned}$$

(a) Construct the TERM set for G .

(b) Use the TERM set to construct an equivalent grammar G_T that does not contain variables that do not generate strings of terminals.

5. (5+10 points) Consider the following grammar G where Σ contains every word listed in the rules: $\Sigma = \{\text{Michigan, Tech, \dots, cool}\}$.

$$\begin{aligned} S &\rightarrow \text{Michigan Tech CS gives } N \mid \text{Having a graduate degree is } R \\ T &\rightarrow \text{Being in a computing field is } D \\ N &\rightarrow \text{BSc degrees} \mid \text{MSc degrees} \mid \text{PhD degrees} \\ R &\rightarrow \text{fun} \mid \text{intellectually challenging} \mid \\ &\quad \text{financially rewarding} \mid \\ &\quad \text{not as hard as one would think} \mid \\ &\quad \text{a worthwhile option to explore} \\ D &\rightarrow \text{fun} \mid \text{awesome} \mid \text{cool} \end{aligned}$$

(a) Construct the REACH set for G .

(b) Use the REACH set to construct an equivalent grammar G_U that does not contain unreachable variables.

6. (15 points) Convert the following grammar G into Chomsky normal form. Show your steps clearly. Note that G already satisfies the conditions on the start symbol S , λ -rules, useless symbols, and chain rules.

$$S \rightarrow bT \quad T \rightarrow aAA \mid AbAT \quad A \rightarrow aT \mid bT \mid a$$