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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. (40 points) Consider the following grammars  $G_1$  and  $G_2$ :

$G_1$  :  
 $S \rightarrow xAx$   
 $A \rightarrow Aa \mid Ab \mid c \mid d$

$G_2$  :  
 $S \rightarrow xAx$   
 $A \rightarrow c \mid d \mid cB \mid dB$   
 $B \rightarrow aB \mid bB \mid a \mid b$

(a) Give a derivation sequence for string  $xcabx$  in  $G_1$ . Give the derivation tree of the sequence.

(b) Give a derivation sequence for string  $xcabx$  in  $G_2$ . Give the derivation tree of the sequence.

2. (60 points) Give a context-free grammar for each of the following languages.

Explain how the grammar works.

**No points will be given if the CFG is not commented.**

(a)  $L = \emptyset$

(b)  $L = \{\lambda\}$

(c)  $L = \{w \mid w \in \{a, b, c\}^*, w \text{ does not contain substring } bc\} = c^*(b \cup ac^*)^*$

(d) ROLL is the language generated by the following recursive definition. It gives a few basic drum roll patterns.  $R$  stands for “right” and  $L$  stands for “left.”

**basis:** “single-stroke:”  $\in$  ROLL, “double-stroke:”  $\in$  ROLL,  
“silly-stroke:”  $\in$  ROLL.

**recursive step:** If ( $w \in$  ROLL and  $w$  contains “single”) then  $wRL$  is in ROLL

If ( $w \in$  ROLL and  $w$  contains “double”) then  $wRLL$  is in ROLL

If ( $w \in$  ROLL and  $w = xy$  where  $x =$  “silly-stroke:”) then  $xRyL$  is in ROLL

**closure:** A string  $w \in$  ROLL only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(e)  $L = \{a^n b^m c^{2n+m} \mid n, m \geq 0\}$

(f)  $L = \{a^n b^m \mid n \neq m, n \geq 0, m \geq 0\}$

(Hint: “not equal to” means “less than or greater than”.)