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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in class. Any other source must be specified clearly.

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**1. (15 points)** Let  $L$  be the language of telephone numbers over  $\Sigma = \{0, 1, 2, 3, (, ), -\}$ .  $\Sigma$  is defined with a subset of the digits 0 to 9 to be able to write short regular expressions without using any extensions. The phone numbers have either of the following two formats and no spaces are allowed. The first digit of the area code or the phone number cannot be zero. Symbol  $n$  represents a digit from  $\Sigma$ . Notice that  $L$  is finite due to the length restriction.

1. Without an area code:  $nnn-nnnn$
2. With an area code:  $(nnn) nnn-nnnn$

- (a) Give a recursive definition for  $L$ .
- (b) Give a regular set for  $L$ .
- (c) Give a regular expression for  $L$ .

**2. (75 points)** Give a regular expression for the following languages.

- (a) The set of strings over  $\{1, 2, 3, a, b, c\}$  that start and end with a number. Strings can have a length of 1 or greater.
- (b) The set of strings over  $\{1, 2, 3, a, b, c\}$  that contain exactly two numbers and the sum of the numbers is even.
- (c) The set of strings over  $\{a, b, c\}$  in which all the  $a$ 's precede the  $b$ 's, which in turn precede the  $c$ 's. It is possible that there are no  $a$ 's, or  $b$ 's, or  $c$ 's and the string is empty.
- (d) The set of strings over  $\{a, b, c\}$  in which all the  $a$ 's precede the  $b$ 's, which in turn precede the  $c$ 's. It is possible that there are no  $a$ 's, or  $b$ 's, or  $c$ 's, but  $\lambda$  is not in the language.
- (e) The set of strings over  $\{1, 2, 3, a\}$  that do not begin with 123.

**3. (15 points)** The following DFA  $M1$  accepts all strings that end in 'ab'. Give a 5-tuple that formally describes the DFA.

