CS3311 Homework 2 Due date: Wednesday, September 14, 2016, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(25 points)* Use induction to prove that $\sum_{i=1}^{n}(2i-1) = n^2$

Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.

2. *(25 points)* Consider the following Python program and use induction on the number of iterations of the `for` loop to prove that the program always prints the following line.
`True:  y is equal to z.`

You must present the proof based on the code and on the number of iterations of the `for` loop. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**. The loop is an implementation of the above question.

```
import operator
n = input('Pick a number greater than 0: ')
print ('You entered %s. \n' % (n))

y=0
for i in range (1,n+1):        # executes for i = 1, 2, ..., n
  x = (2*i) - 1
  y = y+x

z = n**2

if operator.eq (y,z):
  print "True: y is equal to z."
else:
  print "False: y is not equal to z."
```

Please turn the page over for additional questions.

3. *(5+20 points)* Let S_1 be the language generated by the following recursive definition. Assume that the arithmetic computations defined in the recursive step will be performed to obtain the new elements of S_1. The recursive definition is another implementation of the first question.

> **(i) Basis:** $[1, 1] \in S_1$
>
> **(ii) Recursive step:** If $[n, m] \in S_1$, then $[n + 1, m + 2(n + 1) - 1] \in S_1$.
>
> **(iii) Closure:** S_1 consists of exactly the elements that can be obtained by starting with the basis elements of S_1 and applying the recursive step finitely many times to construct new elements of S_1.

(a) Write the first 5 elements of the set S_1 defined recursively.

(b) Use induction to prove for all pairs $[y, z] \in S_1$, $z = y^2$.

You must present the proof based on the recursive definition and on the number of applications of the recursive step. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.

4. *(5+5+5+10 points)* Let L be the language over $\Sigma = \{a, b, d, e\}$ generated by the following recursive definition:

> **basis:** $d \in L$, $e \in L$
>
> **recursive step**: If ($w \in L$ and w contains d) then aaw is in L and wab is in L.
> If ($w \in L$ and w contains e) then $aawab$ is in L.
>
> **closure**: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(a) Give the sets L_1, L_2, and L_3 generated by the recursive definition. Note that $L_0 = \{d, e\}$.

(b) For each of the following five strings, tell whether the string is in L and indicate the reason.

$\lambda, aad, aae, ada, aaeab, abeaa$

(c) Give an implicit definition of the set of strings defined by the recursive definition. An implicit definition describes the pattern of the strings in a set by using a vertical bar to denote "such that". For example: $\{x | x \in \Sigma^*, x = a^i b^i, \text{ and } i \text{ is even }\}$.

(d) Use induction to prove that all the strings in L have an odd length.

You must present the proof based on the recursive definition and on the number of applications of the recursive step. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.