CS3311 Homework 9 — Due date: Wednesday, April 1, 2015, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(5+5 points)* Consider the following grammar G. Note that the grammar does not contain λ-rules except at S.

$$
\begin{aligned}
&S \to aSb \,|\, DEF \,|\, D \,|\, \lambda \\
&D \to E \,|\, EF \,|\, abEF \\
&E \to eEff \,|\, a \,|\, F \\
&F \to ffFe \,|\, a
\end{aligned}
$$

(a) Use algorithm 4.3.1 to construct the `CHAIN` sets for the variables in V.

(b) Construct an equivalent grammar G_c that does not contain chain rules.

2. *(5+5 points)* Consider the following grammar G:

$$
\begin{aligned}
&S \to a \,|\, aA \,|\, BC \\
&A \to aB \,|\, b \\
&B \to Aa \\
&C \to cCD \\
&D \to ddd
\end{aligned}
$$

(a) Construct the TERM set for G.

(b) Use the TERM set to construct an equivalent grammar G_T that does not contain variables that do not generate strings of terminals.

3. *(5+5 points)* Consider the following grammar G where Σ contains every word listed in the rules. $\Sigma = \{$ `Michigan, Tech, ..., cool` $\}$.

$S \to$ `Michigan Tech CS gives` N | `Having a graduate degree is` R
$T \to$ `Being in a computing field is` D
$N \to$ `BSc degrees` | `MSc degrees` | `PhD degrees`
$R \to$ `fun` | `intellectually challenging` |
`financially rewarding` |
`not as hard as one would think` |
`a worthwhile option to explore`
$D \to$ `fun` | `awesome` | `cool`

(a) Construct the REACH set for G.

(b) Use the REACH set to construct an equivalent grammar G_U that does not contain unreachable variables.

4. *(10 points)* Convert the following grammar G into Chomsky normal form. Show your steps clearly. Note that G already satisfies the conditions on the start symbol S, λ-rules, useless symbols, and chain rules.

$$S \to bT \qquad T \to aAA \,|\, AbAT \qquad A \to aT \,|\, bT \,|\, a$$

5. *(10+5 points)* Consider the grammar G from Example 4.5.2:

$$S \to AT \,|\, AB \qquad T \to XB \qquad X \to AT \,|\, AB \qquad A \to a \qquad B \to b$$

(a) Give the upper diagonal matrix produced by the CYK algorithm when run with G and the input string $abbb$. **Show all your work.**

(b) Is $abbb \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

6. *(10+5+10+5+5 points)* Consider the following grammar G. Note that G was obtained by transforming the grammar $S \to aSa \,|\, bSb \,|\, a \,|\, b \,|\, \lambda$ to Chomsky Normal Form.

$$\begin{aligned}
&S \to AR \,|\, BX \,|\, AA \,|\, BB \,|\, a \,|\, b \,|\, \lambda \\
&T \to AR \,|\, BX \,|\, AA \,|\, BB \,|\, a \,|\, b \\
&R \to TA \\
&X \to TB \\
&A \to a \\
&B \to b
\end{aligned}$$

(a) Give the upper diagonal matrix produced by the CYK algorithm when run with G and the input string $abba$. **Show all your work.**

(b) Is $abba \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

(c) Give the upper diagonal matrix produced by the CYK algorithm when run with G and the input string $abbb$. **Show all your work.**

(d) Is $abbb \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

(e) Is $bbb \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

7. *(6+2+2 points)* Consider the following grammar G:

$$\begin{aligned}
&S \to AB \,|\, CB \\
&A \to aaAbb \,|\, aaaAbbb \,|\, Ae \,|\, e \\
&B \to Bd \,|\, d \\
&C \to Cee \,|\, Cff \,|\, gg
\end{aligned}$$

(a) Construct a grammar G' that contains no left-recursive rules and is equivalent to G.

(b) Give a leftmost derivation on the string $aaebb\,ee\,ddd$ in grammar G.

(c) Give a leftmost derivation on the string $aaebb\,ee\,ddd$ in grammar G'.