CS3311 Homework 8

Due date: Wednesday, March 25, 2015, in class

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

1. *(60 points)* Give a context-free grammar for each of the following languages.
Explain how the grammar works.
No points will be given if the CFG is not commented.

(a) $L = \emptyset$

(b) $L = \{\lambda\}$

(c) $L = \{w \,|\, w \in \{a, b, c\}^*, w$ does not contain substring $bc\} = c^*(b \cup ac^*)^*$

(d) ROLL is the language generated by the following recursive definition:

> **basis:** "single-stroke:" $\in$ ROLL, "double-stroke:" $\in$ ROLL,
> "silly-stroke:" $\in$ ROLL.
>
> **recursive step**: If ($w \in$ ROLL and w contains "single") then wRL is in ROLL
> If ($w \in$ ROLL and w contains "double") then $wRRLL$ is in ROLL
> If ($w \in$ ROLL and $w = xy$ where $x =$ "silly-stroke:") then $xRyL$ is in ROLL
>
> **closure**: A string $w \in$ ROLL only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(e) $L = \{a^n b^m c^{2n+m} \mid n, m \geq 0\}$

(f) $L = \{a^n b^m \mid n \neq m, n \geq 0, m \geq 0\}$
(Hint: "not equal to" means "less than or greater than".)

2. *(10 + 5 points)* Consider the following grammar:

$$S \to aSA \,|\, \lambda$$
$$A \to bA \,|\, \lambda$$

(a) Prove the grammar is ambiguous by finding a string that has two distinct leftmost derivations. Show the two derivations.

(b) Build the derivation trees for the derivations in part (a).

Please turn the page over.

3. *(10 + 5 + 10 points)* Consider the following grammar G over $\Sigma = \{a, b\}$.

$$\begin{aligned}&S \to AB \mid aB\\&A \to aB \mid BS\\&B \to bB \mid b \mid \lambda\end{aligned}$$

(a) Transform the grammar into $G1$ so that the recursion to the start symbol is removed.

(b) Show the set of nullable variables in the **new grammar** $G1$.

(c) Construct an essentially noncontracting grammar G_L (with a non-recursive start symbol) equivalent to $G1$. An "essentially noncontracting grammar" is a grammar that has no λ-rules.