CS3311 Homework 2 Due date: Wednesday, February 4, 2015, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(15 points)* Consider the following program segment and use induction on the number of iterations of the `for` loop to prove that the value printed out for `Y` is $\frac{(n^4+2n^3+n^2)}{4}$. You must present the proof based on the pseudocode and on the number of iterations of the `for` loop. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**. The loop is an implementation of: $1 + 8 + 27 \ldots + n^3 = \frac{(n^4+2n^3+n^2)}{4}$.

```
Y = 0;
for I = 1 to n
  {
    Z = (I * I * I);
    Y = Y + Z;
  }
print (Y);
```

2. *(15 points)* Let L be the language over $\Sigma = \{a, b, d, e\}$ generated by the following recursive definition:

> **basis:** $d \in L$, $e \in L$
>
> **recursive step**: If ($w \in L$ and w contains d) then aaw is in L and wab is in L.
> If ($w \in L$ and w contains e) then $aawab$ is in L.
>
> **closure**: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(a) Give the sets L_1, L_2, and L_3 generated by the recursive definition. Note that $L_0 = \{d, e\}$.

(b) For each of the following five strings, tell whether the string is in L and indicate the reason.

$\lambda, aad, aae, ada, aaeab, abeaa$

(c) Give an implicit definition of the set of strings defined by the recursive definition. An implicit definition describes the pattern of the strings in a set by using a vertical bar to denote "such that". For example: $\{x | x \in \Sigma^*$ and x has an even number of a's $\}$

3. *(15 points)* Use induction to prove that all the strings in L above have an odd length.

Please turn the page over for additional questions.

4. *(15 points)* For each of the following regular expressions over $\{a, b\}$, give the minimal length (shortest) string that is **not** in the language defined by the expression.

(a) $(aa)^*(bb)^*a^*$
(b) $a^*(ba)^* \cup b \cup ab \cup aab$
(c) $(a^* \cup b^*)(a^* \cup b^*)(a^* \cup b^*)$

5. *(20 points)* Let L over $\Sigma = \{v, a, r, 1, 2\}$ be the language where every string starts with a number and every v is followed by ar.

(a) Give a recursive definition for L.
(b) Give a regular expression for L.

6. *(20 points)* Give a regular expression for the following languages.

(a) The set of strings over $\{1, 2, 3, a, b, c\}$ that start and end with a number. Consider only strings with length greater than 1.

(b) The set of strings over $\{1, 2, 3, a, b, c\}$ that start with an alphabetical character and contain substring 123.