CS3311 Homework 1 Due date: Wednesday, January 28, 2015, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to write answers in groups. You are allowed to build on the material supplied in class. Any other source must be specified clearly.

Consider the following sets for questions **1** and **2**:

$$X = \{a, b, \{a\}, [a, a]\} \qquad Y = \{a, 99, [a, b], \emptyset, \{\emptyset\}\}$$

1. *(10 points)* Write out each of the sets listed below.

(a) $X \cup Y$

(b) $X \cap Y$

(c) $X - Y$

(d) $P(X)$

(e) $X \times \{a, \emptyset\}$

2. *(10 points)* State whether the following propositions are TRUE or FALSE.

(a) $a \in X$

(b) $\{a\} \in X$

(c) $a \in Y$

(d) $\{a\} \in Y$

(e) $\emptyset \in X$

(f) $\emptyset \in Y$

(g) $\emptyset \subseteq Y$

(h) $\{\emptyset\} \subseteq X$

(f) $\{\emptyset\} \subseteq Y$

(j) $\{[a, a]\} \in X \times X$

3. *(10 points)* Consider a finite set X with n elements where $n \geq 0$. How many elements does $P(X)$– the power set of X– have? Explain your answer.
No points will be given to answers without an accompanying explanation.

Please turn the page over for additional questions.

4. *(15 points)*

(a) Write the first 5 elements of the set S_1 defined recursively. Assume that the arithmetic computation defined in the recursive step will be performed to obtain the new elements of S_1.

> **(i) Basis:** $3 \in S_1$
>
> **(ii) Recursive step:** If $n \in S_1$, then $n + 4 \in S_1$
>
> **(iii) Closure:** S_1 consists of exactly the elements that can be obtained by starting with the basis elements of S_1 and applying the recursive step finitely many times to construct new elements of S_1.

(b) Write the first 10 elements of the set S_2 defined recursively. Assume that the basis elements are strings, and the recursive step performs a string concatenation to obtain new elements.

> **(i) Basis:** $a, b, c \in S_2$
>
> **(ii) Recursive step:** If $w \in S_2$, then $w \times a$, $w \times b$, $w \times c$, w/a, w/b, w/c are $\in S_2$.
>
> **(iii) Closure:** S_2 consists of exactly the elements that can be obtained by starting with the basis elements of S_2 and applying the recursive step finitely many times to construct new elements of S_2.

(c) Write the first 5 elements of the set S_3 defined recursively. Assume that the arithmetic computation defined in the recursive step will be performed to obtain the new elements of S_3.

> **(i) Basis:** $[1, 1] \in S_3$
>
> **(ii) Recursive step:** If $[n, m] \in S_3$, then $[n + 1, m + (n + 1)^2] \in S_3]$.
>
> **(iii) Closure:** S_3 consists of exactly the elements that can be obtained by starting with the basis elements of S_3 and applying the recursive step finitely many times to construct new elements of S_3.

5. *(30 points)* Give a recursive definition for the following sets. You may use the addition or other operators to generate new integer elements. There are no restrictions on the conditions checked via an `if` statement. Explain the "pattern" you are using.
No points will be given to answers without an accompanying explanation.

(a) $\{3, 7, 15, 31, \ldots\}$

(b) $\{3, 7, 15, 31, \ldots\} \cup \{4, 8, 12, 16, \ldots\}$

(c) $\{[1, aa], [2, bbb], [3, aaaa], [4, bbbbbb], [5, a^6], [6, b^9], \ldots\}$

6. *(3+7 points)* Consider the following infinite loop in pseucocode that fills out a set C.
Note that $\leftarrow$ is used as the assignment operator, i and k are integers, and j is a string. For i, an integer, "+" represents addition. For j, a string, "+" represents concatenation. For k, an integer, "-" represents subtraction.

$C \leftarrow \emptyset$
$i \leftarrow 3$
$j \leftarrow$'learn to like '
$k \leftarrow -1$
while (true) do {
    $C \leftarrow C \cup \{i\} \cup \{j\} \cup \{k\}$
    $i \leftarrow i + 3$
    $j \leftarrow j+$ ' winter'
    $k \leftarrow 2k - k$
}

(a) List the elements of set C after 0, 1, and 2 iterations of the loop.

(b) Give a recursive definition for the set C.

7. *(15 points)* Use induction to prove that $\sum_{i=1}^{n} i^3 = \frac{(n^4+2n^3+n^2)}{4}$.

Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.