The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

1. *(10+5 points)* Consider the grammar G from Example 4.5.2:

$$S \rightarrow AT \mid AB$$
$$T \rightarrow XB$$
$$X \rightarrow AT \mid AB$$
$$A \rightarrow a$$
$$B \rightarrow b$$

(a) Give the upper diagonal matrix produced by the CYK algorithm when run with G and the input string $abbb$. **Show all your work.**

(b) Is $abbb \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

2. *(10+5+10+5+5 points)* Consider the following grammar G. Note that G was obtained by transforming the grammar $\quad S \rightarrow aSa \mid bSb \mid a \mid b \mid \lambda \quad$ to Chomsky Normal Form.

$$S \rightarrow AR \mid BX \mid AA \mid BB \mid a \mid b \mid \lambda$$
$$T \rightarrow AR \mid BX \mid AA \mid BB \mid a \mid b$$
$$R \rightarrow TA$$
$$X \rightarrow TB$$
$$A \rightarrow a$$
$$B \rightarrow b$$

(a) Give the upper diagonal matrix produced by the CYK algorithm when run with G and the input string $abba$. **Show all your work.**

(b) Is $abba \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

(c) Give the upper diagonal matrix produced by the CYK algorithm when run with G and the input string $abbb$. **Show all your work.**

(d) Is $abbb \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

(e) Is $bbb \in L(G)$? Why? Provide the reason based on the upper diagonal matrix you constructed.

Please turn the page over for additional questions.

3. *(10+5+5 points)* Consider the following grammar G:

$$S \rightarrow AB \,|\, CB$$
$$A \rightarrow aaAbb \,|\, aaaAbbb \,|\, Ae \,|\, e$$
$$B \rightarrow Bd \,|\, d$$
$$C \rightarrow Cee \,|\, Cff \,|\, gg$$

(a) Construct a grammar G' that contains no left-recursive rules and is equivalent to G.

(b) Give a leftmost derivation on the string $aaebb\,ee\,ddd$ in grammar G.

(c) Give a leftmost derivation on the string $aaebb\,ee\,ddd$ in grammar G'.

4. *(30 points)* Consider the following grammar.

$$S \rightarrow ABab \,|\, BAba$$
$$A \rightarrow a \,|\, c$$
$$B \rightarrow b \,|\, c \,|\, \lambda$$

(a) Draw the graph of the above grammar.

(b) Give the lookahead sets for each variable and rule.

(c) What is the lookahead length needed to deterministically parse strings from this grammar? Explain your answer.