1. (10+ 10+ 10 points) Consider the following grammar $G$ over $\Sigma = \{a, b\}$.

   $S \rightarrow AB | aB$
   $A \rightarrow aB | BS$
   $B \rightarrow bB | b | \lambda$

   (a) Transform the grammar into $G_1$ so that the recursion to the start symbol is removed.

   (b) Show the set of nullable variables in the new grammar $G_1$.

   (c) Construct an essentially noncontracting grammar $G_L$ (with a non-recursive start symbol) equivalent to $G_1$.

2. (10+10 points) Consider the following grammar $G$. Note that the grammar does not contain $\lambda$-rules except at $S$.

   $S \rightarrow aSb | DEF | D | \lambda$
   $D \rightarrow E | EF | abEF$
   $E \rightarrow eEF | a | F$
   $F \rightarrow fF | a$

   (a) Use algorithm 4.3.1 to construct the CHAIN sets for the variables in $V$.

   (b) Construct an equivalent grammar $G_c$ that does not contain chain rules.

3. (10+10 points) Consider the following grammar $G$:

   $S \rightarrow a | aA | BC$
   $A \rightarrow aB | b$
   $B \rightarrow Aa$
   $C \rightarrow cCD$
   $D \rightarrow ddd$

   (a) Construct the TERM set for $G$.

   (b) Use the TERM set to construct an equivalent grammar $G_T$ that does not contain variables that do not generate strings of terminals.

   Please turn the page over for additional questions.
4. (10+10 points) Consider the following grammar $G$ with $\Sigma = \{ \text{Robots, Humans, have, batteries, computers, artificial intelligence, food, brains, natural intelligence, Working, with, computers, is, fun, awesome, cool} \}$.

$$
S \rightarrow \text{Robots have } N \mid \text{Humans have } R \\
T \rightarrow \text{Working with computers is } D \\
N \rightarrow \text{batteries } \mid \text{computers } \mid \text{artificial intelligence} \\
R \rightarrow \text{food } \mid \text{brains } \mid \text{natural intelligence} \\
D \rightarrow \text{fun } \mid \text{awesome } \mid \text{cool}
$$

(a) Construct the REACH set for $G$.

(b) Use the REACH set to construct an equivalent grammar $G_U$ that does not contain unreachable variables.

5. (10 points) Convert the following grammar $G$ into Chomsky normal form. Show your steps clearly. Note that $G$ already satisfies the conditions on the start symbol $S$, $\lambda$-rules, useless symbols, and chain rules.

$$
S \rightarrow bT \\
T \rightarrow aAA \mid AbAT \\
A \rightarrow aT \mid bT \mid a
$$