CS3311 Homework 8 Due date: Wednesday, March 26, 2014, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(20 points)* Consider the following grammars G_1 and G_2:

G_1 :
$S \rightarrow xAx$
$A \rightarrow Aa \,|\, Ab \,|\, c \,|\, d$

G_2 :
$S \rightarrow xAx$
$A \rightarrow c \,|\, d \,|\, cB \,|\, dB$
$B \rightarrow aB \,|\, bB \,|\, a \,|\, b$

(a) Give a derivation sequence for string $xcabx$ in G_1. Give the derivation tree of the sequence.

(b) Give a derivation sequence for string $xcabx$ in G_2. Give the derivation tree of the sequence.

2. *(5+5+10 points)* Consider the following grammar G_1:

$S \rightarrow aSb \,|\, A$
$A \rightarrow cA \,|\, d \,|\, \lambda$

(a) Give a derivation for a **terminal string** such that the $S \rightarrow aSb$ rule is used exactly twice, the $A \rightarrow cA$ rule is used exactly three times, and the $A \rightarrow d$ rule is used once during the derivation.

(b) Give a derivation for a **terminal string** such that the $S \rightarrow aSb$ rule is used exactly once, the $A \rightarrow cA$ rule is also used exactly twice, and the $A \rightarrow \lambda$ rule is used once during the derivation.

(c) Use set notation to define the language generated by the grammar.

3. *(40 points)* Give a context-free grammar for each of the following languages. Explain how the grammar works.

(a) $(a \cup b)^*(b \cup aa)(a \cup b)^*$

(b) $L = \{a^n b^m c^{2n+m} \mid n, m \geq 0\}$

(c) $L = \{a^m b^i a^n \mid i = m + n, n \geq 0, m \geq 0\}$

(d) $L = \{a^n b^m \mid n \neq m, n \geq 0, m \geq 0\}$
(Hint: "not equal to" means "less than or greater than".)

Please turn the page over.

4. *(40 points)* Let $G = (V = \{S, A, B\}, \Sigma = \{a, b\}, P, S)$ where P are the following:

$$S \to BAS \mid \lambda$$
$$A \to aA \mid a$$
$$B \to bB \mid \lambda$$

(a) Give a leftmost derivation for $bbabaa$.

(b) Give a regular expression for $L(G)$.

(c) Prove that the grammar is ambiguous by giving two distinct leftmost derivations of aa.

(d) Build the derivation trees for the derivations in part (c).