1. (20 points) For each of the following regular expressions over \{a, b\}, give the minimal length (shortest) string that is not in the language defined by the expression.

(a) \(a^* (ba)^* \cup b \cup ab \cup aab\)
(b) \((a^* \cup b^*) (a^* \cup b^*) (a^* \cup b^*)\)

2. (30 points) Let \(L\) over \(\Sigma = \{a, b\}\) be the language where every string contains at least one ‘b’ and has an even number of ‘a’s before the first ‘b’. Take zero as an even number.

(a) Give a recursive definition for \(L\).
(b) Give a regular set for \(L\).
(c) Give a regular expression for \(L\).

3. (50 points) Give a regular expression for the following languages.

(a) The set of strings over \(\{1, 2, 3, a, b, c\}\) that start and end with a number. Consider only strings with length greater than 1.
(b) The set of strings over \(\{1, 2, 3, a, b, c\}\) that start with an alphabetical character and contain substring 123.
(c) The set of strings over \(\{1, 2, 3, a, b, c\}\) that contain exactly two numbers and the sum of the numbers is even.
(d) The set of strings over \(\{1, 2, 3, a\}\) that do not begin with 123.
(e) The set of strings over \(\{a, b, c\}\) in which all the a’s precede the b’s, which in turn precede the c’s. It is possible that there are no a’s, or b’s, or c’s, but \(\lambda\) is not in the language.