CS3311 Homework 2

Due date: Wednesday, January 29, 2014, in class

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(20 points)* Let X be an infinite set. Give a recursive definition of Y, the set of pairs such that the members of the pair come from X and they are the same. In other words,

$Y = \{[x, x] \mid x \in X\}$.

For example, if X is $\{0, 1, 2, 3, \ldots\}$ then $Y = \{[0, 0], [1, 1], [2, 2], [3, 3], \ldots\}$.

Assume that there is an already defined function `NEXT-ELEMENT(a)` that gives the element in X that follows a according to some ordering. You don't need to specify the ordering. You may also assume a function `FIRST-ELEMENT()` that returns the first element of X.

2. *(20 points)* Use induction to prove that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$.

Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.

3. *(20 points)* Consider the following program segment and use induction on the number of iterations of the `for` loop to prove that the value printed out for `Y` is $\frac{n}{n+1}$. You must present the proof based on the pseudocode and on the number of iterations of the `for` loop. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**. Note that the loop is an implementation of the summation given in the previous question.

```
Y = 0;
for I = 1 to n
  {
    Z = (1 / (I * (I + 1)));
    Y = Y + Z;
  }
print (Y);
```

Please turn the page over.

4. *(40 points)* Let L be the language over $\Sigma = \{a, b, +, (,)\}$ generated by the following recursive definition:

> **basis:** $a \in L$
>
> **recursive step**: If $(w \in L)$ then $w + b$ is in L, and (w) is in L.
>
> **closure**: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(a) Give the sets L_1, L_2, and L_3 generated by the recursive definition. Note that $L_0 = \{a\}$.

(b) For each of the following strings, tell whether the string is in L and indicate the reason.

λ
a
$a + b$
$a + b + b$
(a)
(b)
$a + (b)$
$(a) + b$
$(a + b + b)$
$((a + b) + b)$