
The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to write answers in groups. You are allowed to build on the material supplied in class. Any other source must be specified clearly.

Consider the following sets for questions 1 and 2: $X = \{1, b, \emptyset\}$ $Y = \{a, b, c, 2, \emptyset, \{a\}, \{\emptyset\}\}$

1. (20 points) Write out each of the sets listed below.

- | | | | |
|----------------------------|----------------------------------|------------------------|------------------------|
| (a) $X \cup Y$ | (b) $X \cap Y$ | (c) $X \cup \emptyset$ | (d) $Y \cap \emptyset$ |
| (e) $X \cap \{\emptyset\}$ | (f) $X - Y$ | (g) $Y - X$ | (h) $P(X)$ |
| (i) $X \times Y$ | (j) $X \times Y \cap \{[a, a]\}$ | | |

2. (20 points) State whether the following propositions are TRUE or FALSE.

- | | | | |
|-----------------------------|---------------------------------|-----------------------|-----------------------|
| (a) $b \in X$ | (b) $\{b\} \in X$ | (c) $b \in Y$ | (d) $\{b\} \in Y$ |
| (e) $\{b\} \subseteq X$ | (f) $\{b\} \subseteq Y$ | (g) $\emptyset \in X$ | (h) $\emptyset \in Y$ |
| (i) $\emptyset \subseteq Y$ | (j) $\{[b, b]\} \in X \times X$ | | |

3. (20 points) Consider a finite set X with n elements where $n \geq 0$. How many elements does $P(X)$ —the power set of X —have? Explain your answer. No points will be given to answers without an accompanying explanation.

4. (20 points) Consider the following infinite loop that fills out a set C . Note that \leftarrow is used as the assignment operator, i is an integer, and j is a string. For i , an integer, “+” represents addition. For j , a string, “+” represents concatenation. Variable k is also an integer.

```

C ← ∅
i ← 1
j ← 'ab'
k ← -3
while (true) do {
    C ← C ∪ {i} ∪ {j} ∪ {k}
    i ← i + 2
    j ← j + 'ab'
}

```

List the elements of C after 0, 1, and 2 iterations of the loop.

Give a recursive definition for the set C .

Please turn the page over.

5. (20 points) Give a recursive definition of the following sets. You may only use the successor operator to generate new integer elements. However, there are no restrictions on the conditions checked via an `if` statement. For part (a), list a few elements of the set. For part (b), some members are already listed, list additional elements of the set.

Part a. $S_1 = \{k \mid k \in \mathbb{N} \text{ and } (k \text{ is even or divisible by } 3)\}$

Part b. $S_2 = \{1, 3, 5, 7, \dots\} \cup \{[1, a], [3, aaa], [5, aaaaa], \dots\}$