CS3311 Homework 8 — Due date: Wednesday, October 29, 2014, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(60 points)* Give a context-free grammar for each of the following languages.
Explain how the grammar works.
No points will be given if the CFG is not commented.

(a) $L = \emptyset$

(b) $L = \{\lambda\}$

(c) $L = \{w \mid w \in \{a, b, c\}^*, w$ does not contain substring $bc\} = c^*(b \cup ac^*)^*$

(d) L is the language over $\Sigma = \{a, b, d, e\}$ generated by the following recursive definition:

> **basis:** $d \in L, e \in L$
>
> **recursive step**: If ($w \in L$ and w contains d) then aaw is in L and wab is in L.
> If ($w \in L$ and w contains e) then $aawab$ is in L.
>
> **closure**: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(e) $L = \{a^n b^m c^{2n+m} \mid n, m \geq 0\}$

(f) $L = \{a^n b^m \mid n \neq m, n \geq 0, m \geq 0\}$
(Hint: "not equal to" means "less than or greater than".)

2. *(40 points)* Let $G = (V = \{S, A, B\}, \Sigma = \{a, b\}, P, S)$ where P are the following:

$$S \rightarrow BAS \mid \lambda$$
$$A \rightarrow aA \mid a$$
$$B \rightarrow bB \mid \lambda$$

(a) Give a leftmost derivation for $bbabaa$.

(b) Give a regular expression for $L(G)$.

(c) Prove that the grammar is ambiguous by giving two distinct leftmost derivations of aa.

(d) Build the derivation trees for the derivations in part (c).