CS3311 Homework 2

Due date: Wednesday, September 17, 2014, in class

The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(20 points)* Let X be an infinite set. Assume that there are two already defined functions that return elements of this set. The function `FIRST-ELEMENT()` returns the first element of X. The function `NEXT-ELEMENT(a)` returns the element in X that follows a according to some ordering.

Give a recursive definition of Y, the subset of X that contains every other element of X, starting with the first element of X.

For example, if X is $\{1, a, b, c, 2, aa, bb, cc, 3, \ldots\}$ then $Y = \{1, b, 2, bb, 3, \ldots\}$.

2. *(15 points)* Use induction to prove that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**.

3. *(15 points)* Consider the following program segment and use induction on the number of iterations of the `for` loop to prove that the value printed out for `Y` is $\frac{n(n+1)(2n+1)}{6}$. You must present the proof based on the pseudocode and on the number of iterations of the `for` loop. Clearly label the **basis**, **inductive hypothesis**, and **inductive step**. The loop is an implementation of: $1 + 4 + 9 + 16 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

```
Y = 0;
for I = 1 to n
  {
    Z = (I * I);
    Y = Y + Z;
  }
print (Y);
```

Please turn the page over for additional questions.

4. *(20 points)* Let L be the language over $\Sigma = \{a, b, d, e\}$ generated by the following recursive definition:

basis: $d \in L$, $e \in L$

recursive step: If ($w \in L$ and w contains d) then aaw is in L and wab is in L.
If ($w \in L$ and w contains e) then $aawab$ is in L.

closure: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

(a) Give the sets L_1, L_2, and L_3 generated by the recursive definition. Note that $L_0 = \{d, e\}$.

(b) For each of the following five strings, tell whether the string is in L and indicate the reason.

$\lambda, aad, aae, ada, aaeab, abeaa$

(c) Give an implicit definition of the set of strings defined by the recursive definition. An implicit definition describes the pattern of the strings in a set by using a vertical bar to denote "such that". For example: $\{x|x \in \Sigma^*$ and x has an even number of a's $\}$

5. *(15 points)* Use induction to prove that all the strings in L above have an odd length.

6. *(15 points)* For each of the following regular expressions over $\{a, b\}$, give the minimal length (shortest) string that is **not** in the language defined by the expression.

(a) $(aa)^*(bb)^*a^*$
(b) $a^*(ba)^* \cup b \cup ab \cup aab$
(c) $(a^* \cup b^*)(a^* \cup b^*)(a^* \cup b^*)$