The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

1. *(60 points)* Give a context-free grammar for each of the following languages. Explain how the grammar works.

(a) $(a \cup b)^* a$

(b) $(a \cup b)^* (b \cup aa)(a \cup b)^*$

(c) The set of even-length strings in $\{a, b\}$ with the two middle symbols equal. (λ is not in the language.)

(d) $L = \{a^n b^m c^{2n+m} \mid n, m \geq 0\}$

(e) $L = \{a^m b^i a^n \mid i = m + n, n \geq 0, m \geq 0\}$

(f) $L = \{a^n b^m \mid n \neq m, n \geq 0, m \geq 0\}$
(Hint: "not equal to" means "less than or greater than".)

2. *(40 points)* Let $G = (V = \{S, A, B\}, \Sigma = \{a, b\}, P, S)$ where P are the following:

$$S \rightarrow BAS \mid \lambda$$
$$A \rightarrow aA \mid a$$
$$B \rightarrow bB \mid \lambda$$

(a) Give a leftmost derivation for $bbabaa$.

(b) Give a regular expression for $L(G)$.

(c) Prove that the grammar is ambiguous by giving two distinct leftmost derivations of aa.

(d) Build the derivation trees for the derivations in part (c).