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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. (5+10 points) Consider the following grammar  $G$ :

$$\begin{aligned} S &\rightarrow AB \mid DE \mid AfB \mid DfE \\ A &\rightarrow aA \mid B \\ B &\rightarrow bB \mid b \\ D &\rightarrow dD \mid E \\ E &\rightarrow eE \mid D \end{aligned}$$

**Part a.** Construct the TERM set for  $G$ .

**Part b.** Use the TERM set to construct an equivalent grammar  $G_T$  that does not contain variables that do not generate strings of terminals.

2. (5+10 points) Consider the following grammar  $G$  with  $\Sigma = \{ \text{is, are, the, trees, rainbow, game, yellow, orange, red, purple, beautiful, breathtaking, bright, fun, enjoyable, lively} \}$ .

$$\begin{aligned} S &\rightarrow T_1 \text{ are } C \mid T_1 \text{ are } A \\ O &\rightarrow T_2 \text{ is } A \mid T_3 \text{ is } B \\ T_1 &\rightarrow \text{the trees} \\ T_2 &\rightarrow \text{the rainbow} \\ T_3 &\rightarrow \text{the game} \\ A &\rightarrow \text{beautiful} \mid \text{breathtaking} \mid \text{bright} \\ B &\rightarrow \text{fun} \mid \text{enjoyable} \mid \text{lively} \\ C &\rightarrow \text{yellow} \mid \text{orange} \mid \text{purple} \mid \text{red} \end{aligned}$$

**Part a.** Construct the REACH set for  $G$ .

**Part b.** Use the REACH set to construct an equivalent grammar  $G_U$  that does not contain unreachable variables.

3. (20 points) Convert the following grammar  $G$  into Chomsky normal form. Show your steps clearly. Note that  $G$  already satisfies the conditions on the start symbol  $S$ ,  $\lambda$ -rules, useless symbols, and chain rules.

$$\begin{aligned} S &\rightarrow ABDA \\ A &\rightarrow ab \\ B &\rightarrow bB \mid b \\ D &\rightarrow dDd \mid eDe \mid b \end{aligned}$$

4. (20 points) Consider the grammar  $G$  from Example 4.5.2:

$$\begin{aligned} S &\rightarrow AT \mid AB \\ T &\rightarrow XB \\ X &\rightarrow AT \mid AB \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

Give the upper diagonal matrix produced by the CYK algorithm when run with  $G$  and the input string  $abb$ . Is  $abb \in L(G)$ ? Why? Provide the reason based on the upper diagonal matrix you constructed.

5. (20+5+5 points) Consider the following grammar  $G$ . Note that  $G$  was obtained by transforming the grammar  $S \rightarrow bSb \mid a$  to Chomsky Normal Form.

$$\begin{aligned} S &\rightarrow BR \mid a \\ T &\rightarrow BR \mid a \\ R &\rightarrow TB \\ B &\rightarrow b \end{aligned}$$

**Part a.** Give the upper diagonal matrix produced by the CYK algorithm when run with  $G$  and the input string  $bbabb$ . **Show all your work.**

**Part b.** Is  $bbabb \in L(G)$ ? Why? Provide the reason based on the upper diagonal matrix you constructed.

**Part c.** Is  $bba \in L(G)$ ? Why? Provide the reason based on the upper diagonal matrix you constructed.