
The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

1. (15 points) Consider the following grammar G :

$$S \rightarrow Sab \mid AB$$

$$A \rightarrow dAee \mid \lambda$$

$$B \rightarrow fBg \mid \lambda$$

Part a. Give a derivation for a terminal string such that the $S \rightarrow Sab$ rule is used exactly twice, the $A \rightarrow dAee$ rule is used exactly once, and the $B \rightarrow fBg$ rule is used exactly once during the derivation.

Part b. Draw the derivation tree for the above derivation.

Part c. Use set notation to define the language generated by the grammar.

2. (20 points) For each of the following languages: Give a regular expression that represents the language, and give a context free grammar that generates the language.

Part a. $L = \{w \mid w \in \{a, b\}^* \text{ and the substring } aa \text{ occurs exactly once in } w\}$

Part b. $L = \{w \mid w \in \{a, b\}^* \text{ and the number of } a\text{'s in } w \text{ is divisible by } 3\}$

3. (15 points) Construct a **context-free grammar** over $\{a, b, c\}$ for the language $\{a^n b^{m+2n} c^m \mid n, m \geq 0\}$. Explain how you construct the grammar.

4. (20 points) Consider the CFG G defined by the following productions. Prove by induction that every string in $L(G)$ has ab as a substring.

$$S \rightarrow aAb$$

$$A \rightarrow aA \mid bA \mid \lambda$$

5. (30 points) Consider the following grammar: $S \rightarrow aaS \mid aaaS \mid \lambda$

a. Give a regular expression or a set-theoretic definition for the language of the grammar.

b. Show that the grammar is ambiguous.

c. Construct an equivalent unambiguous grammar. Explain how you construct the grammar.