CS3311 Homework 4 Due date: Wednesday, October 3, 2012, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(20 points)* Let L be the language over $\{a, b\}$ generated by the following recursive definition

basis: $\lambda \in L$
recursive step: If $w \in L$ then wab is in L.
closure: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

Part a. Give the sets L_1, L_2, and L_3 generated by the recursive definition. Note that $L_0 = \{\lambda\}$.

Part b. Give an implicit definition of the set of strings defined by the recursive definition. An implicit definition describes the pattern of the strings in a set by using a vertical bar to denote "such that". For example: $\{x|x \in \Sigma^*$ and x has an even number of a's $\}$

2. *(25 points)* Consider a set W defined by the following recursive definition:

basis: $W = \{d\}$
recursive step: If $w \in W$ then wab is in W and bcw is in W.
closure: A string $w \in W$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

Part a. Give the sets L_0 and L_1 generated by the recursive definition.

Part b. Give the set L_2 generated by the recursive definition.

Part c. Is $bcdab \in W$? Why?

Part d. Is $bcbcbcdababab \in W$? Why?

Part e. Is $abababab dbc \in W$? Why?

3. *(25 points)* Use induction to prove that all the strings in W above have an odd length.

4. *(30 points)* Give recursive definitions for the following languages:

Part a. N_S: The set of strings representing the natural numbers.
Part b. $M = \{a^i b^j | i > j \geq 0\}$