CS3311 Homework 2 Due date: Wednesday, September 19, 2012, in class

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The answers must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. Any other source must be specified clearly.

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1. *(30 points)* Let X, Y and Z be the following sets.
$X = \{$`Chicago Tribune, Daily Mining Gazette, Detroit Free Press, Star Tribune, Wisconsin State Journal`$\}$
$Y = \{$`Paul Anger, Nancy Barnes, Gerould W. Kern, John Smalley`$\}$
$Z = \{$`Chicago, Detroit, Green Bay, Houghton, Madison, Minneapolis`$\}$

The unary functions $f : X \to Y$ and $g : X \to Z$ are described in the following tables.

Newspaper n	Editor f(n)	City g(n)
Chicago Tribune	Gerould W. Kern	Chicago
Daily Mining Gazette		Houghton
Detroit Free Press	Paul Anger	Detroit
Star Tribune	Nancy Barnes	Minneapolis
Wisconsin State Journal	John Smalley	Madison

a. What are the values of f(`Star Tribune`), f(`Daily Mining Gazette`), and $g(f^{-1}$(`Paul Anger`)$)$?
b. What are the domain, co-domain, and range of f?
c. What are the domain, co-domain, and range of g?
d. Is f total, one-to-one, or onto? Is g total, one-to-one, or onto?
e. Define a new set W. Write a function $h : X \to W$ such that h is total and onto, but not one-to-one. You may use a table to describe the function.

2. *(30 points)* Suppose that X and Y are finite, non-empty sets and X has n elements. Consider a function f such that $f : X \to Y$. What can be said about the cardinality of Y for the following cases? Explain your answer.

a. Function f is one-to-one.
b. Function f is onto.
c. Function f is one-to-one and onto.

Please turn the page over.

3. *(20 points)*
a. Consider the set $A = \{k \mid k \in \mathbb{N}$ and the most significant digit of k is $1\}$. The set A contains only those numbers that start with 1 and thus A is clearly a proper subset of $\mathbb{N}$, i.e., $A \subset \mathbb{N}$. Write A using the "dot-dot" notation, i.e., as a list that ends with "$\ldots$" Show that A and $\mathbb{N}$ have the same cardinality. In other words, prove that A is denumerable.

b. Consider the set $B = \mathbb{N} \cup \{x + 0.1 \mid x \in \mathbb{N}\}$. Clearly, B is larger than $\mathbb{N}$, i.e., $\mathbb{N} \subset B$. Write B using the "dot-dot" notation, i.e., as a list that ends with "$\ldots$" Show that B and $\mathbb{N}$ have the same cardinality. In other words, prove that B is denumerable.

4. *(20 points)* Assume that the set S is infinite and countable, and the set T is finite. Prove that $S \times T$ is countable.