
The answers, comments, and programs (if any) must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. If you use any other source than the class notes and the textbook, specify it clearly.

1. (15 points) Consider the CFG G defined by the following productions. Prove by induction that every string in $L(G)$ has ab as a substring.

$$\begin{aligned} S &\rightarrow aAb \\ A &\rightarrow aA \mid bA \mid \lambda \end{aligned}$$

2. (15 points) Consider the following grammar: $S \rightarrow aaS \mid aaaS \mid \lambda$

- Give a regular expression or a set-theoretic definition for the language of the grammar.
- Show that the grammar is ambiguous.
- Construct an equivalent unambiguous grammar. Explain how you construct the grammar.

3. (15 points) Consider the following grammar:

$$\begin{aligned} S &\rightarrow aSA \mid \lambda \\ A &\rightarrow bA \mid \lambda \end{aligned}$$

- Give a regular expression or a set-theoretic definition for the language of the grammar.
- Show that the grammar is ambiguous.
- Construct an equivalent unambiguous grammar. Explain how you construct the grammar.

4. (40 points) Give a context-free grammar for each of the following languages. Briefly explain how you construct the grammar.

- $L = \{a^n b^{2n} c^m \mid n, m \geq 0\}$
- $L = \{a^n b^m c^{2n+m} \mid n, m \geq 0\}$
- $L = \{a^m b^i a^n \mid i = m + n, n \geq 0, m \geq 0\}$
- $L = \{a^n b^m c^k \mid n = m \text{ or } m = k \text{ where } n, m, k \geq 0\}$

Please turn the page over.

5. (2+3+10 points) Consider the following grammar:

$$S \rightarrow BSA \mid A$$

$$A \rightarrow aA \mid \lambda$$

$$B \rightarrow Bba \mid \lambda$$

- a. Transform the grammar into G_1 so that the recursion to the start symbol is removed.
- b. Show the set of nullable variables in G_1 .
- c. Construct an essentially noncontracting grammar G_2 (with a non-recursive start symbol) equivalent to G_1 .