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The answers, comments, and programs (if any) must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. If you use any other source than the class notes and the textbook, specify it clearly.

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1. (15 points) For each of the following regular expressions, give minimal length strings that are not in the language defined by the expression.

- a.  $a^*(abb)^* \cup b$
- b.  $(a^* \cup b^*)(a^* \cup b^*)(a^* \cup b^*)$
- c.  $b^*(a \cup ba)^*b^*$

2. (15 points) Let  $L$  over  $\Sigma = \{a, b\}$  be the language where each string ends with an  $a$ .

- a. Give a recursive definition for  $L$ .
- b. Give a regular set for  $L$ .
- c. Give a regular expression for  $L$ .

3. (15 points) Let  $L$  over  $\Sigma = \{a, b, c\}$  be the language where every  $b$  is followed by  $aa$ .

- a. Give a recursive definition for  $L$ .
- b. Give a regular set for  $L$ .
- c. Give a regular expression for  $L$ .

4. (30 points) Give a regular expression for the following languages.

- a. The set of strings over  $\{a, b, c\}$  with length three.
- b. The set of strings over  $\{a, b, c\}$  with length greater than three.
- c. The set of strings over  $\{a, b, c\}$  where the total number of  $b$ 's and  $c$ 's is three.
- d. The set of strings over  $\{a, b, c\}$  in which all the  $a$ 's precede the  $b$ 's, which in turn precede the  $c$ 's. It is possible that there are no  $a$ 's, or  $b$ 's, or  $c$ 's.
- e. The set of strings over  $\{a, b, c\}$  in which all the  $a$ 's precede the  $b$ 's, which in turn precede the  $c$ 's. It is possible that there are no  $a$ 's, or  $b$ 's, or  $c$ 's, but  $\lambda$  is not in the language.
- f. The set of strings over  $\{a, b, c\}$  that do not begin with the substring  $aaa$ .

*Please turn the page over for the remaining questions.*

5. (20 points) Let  $G = (V = \{S, A, B\}, \Sigma = \{a, b\}, P, S)$  where  $P$  are the following:

$$S \rightarrow SAB \mid \lambda$$

$$A \rightarrow aA \mid a$$

$$B \rightarrow bB \mid \lambda$$

- a. Give a leftmost derivation for  $abbaab$ .
- b. Give two distinct leftmost derivations of  $aa$ .
- c. Build the derivation trees for the derivations in part (b).
- d. Give a regular expression for  $L(G)$ .

6. (5 points) How many different derivations are there that generate the following derivation tree? Consider every ordering of variable replacement as a different derivation.

