CS3311 Homework 4 — Due date: Wednesday, September 29, 2010, in class

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The answers, comments, and programs (if any) must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. If you use any other source than the class notes and the textbook, specify it clearly.

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1. *(25 points)* Consider a set W defined by the following recursive definition:

basis: $W = \{0, 1, 2, \ldots, 9\}$

recursive step: If $w \in W$ then (w) is in W. If $w_1 \in W$ and $w_2 \in W$ then $w_1 + w_2 \in W$.

closure: A string $w \in W$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

Part a. Is $1000 \in W$? Why?

Part b. Is $1 + 00 \in W$? Why?

Part c. Is $1 + 0 + 0 \in W$? Why?

Part d. Is $((1)) \in W$? Why?

Part e. Is $(1 + 9) \in W$? Why?

2. *(20 points)* Use induction to prove that all the strings in W above have an odd length.

3. *(5+5+10 points)* Let L be the language over $\{:,)\}$ generated by the following recursive definition

basis: $\lambda \in L$

recursive step: If $w \in L$ then $: w))$ is in L.

closure: A string $w \in L$ only if it can be obtained from the basis set by a finite number of applications of the recursive step.

Part a. Give the sets L_0, L_1, and L_2 generated by the recursive definition.

Part b. Give an implicit definition of the set of strings defined by the recursive definition.

Part c. Prove by mathematical induction that for every string u in L, the number of $)$'s in u is twice the number of :'s in u. Let $n_p(u)$ and $n_c(u)$ denote the number of $)$'s and the number of :'s in u, respectively.

4. *(15 points)* Give a recursive definition of the length of a string over Σ. Use the primitive operation from the definition of string.

5. *(20 points)* Prove the following by induction: $(w^R)^R = w, \forall w \in \Sigma^*$.