

The answers, comments, and programs (if any) must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. If you use any other source than the class notes and the textbook, specify it clearly.

**1. (25 points)** Let  $X$  be the set  $\{1, 2, 3, 4, 5\}$  and  $Y$  be the set  $\{6, 7, 8, 9, 10\}$ . The unary function  $f : X \rightarrow Y$  and the binary function  $g : X \times Y \rightarrow Y$  are described in the following tables.

n	f (n)	g	6	7	8	9	10
1	7	1	7	8	9	10	6
2	6	2	10	10	10	10	10
3	7	3	7	7	8	8	9
4	6	4	9	8	7	6	10
5	7	5	6	6	6	6	6

- a.** What are the values of  $f(2)$  and  $g(4, f(4))$ ?
  - b.** What are the range, co-domain, and domain of  $f$ ?
  - c.** What are the range, co-domain, and domain of  $g$ ?
  - d.** Is  $f$  one-to-one, onto, or total?
  - e.** Write a function  $h : X \rightarrow Y$  such that  $h$  is one-to-one, onto, and total. You may use a table to describe the function.
- 2. (30 points)** Suppose that  $X$  and  $Y$  are finite, non-empty sets and  $X$  has  $n$  elements. Consider a function  $f$  such that  $f : X \rightarrow Y$ . What can be said about the cardinality of  $Y$  for the following cases?
- a.** Function  $f$  is one-to-one.
  - b.** Function  $f$  is onto.
  - c.** Function  $f$  is one-to-one and onto.
- 3. (20 points)** Prove that the set of odd negative integers is denumerable.
- 4. (25 points)** Assume that the set  $S$  is infinite and countable, and the set  $T$  is finite. Prove that  $S \cup T$  is countable.