
The answers, comments, and programs (if any) must be the original work of the author. While discussion with others is permitted and encouraged, the final work should be done individually. You are not allowed to work in groups. You are allowed to build on the material supplied in the class. If you use any other source than the class notes and the textbook, specify it clearly.

Consider the following sets for questions **1** and **2**:

$$X = \{a, b, c\} \quad Y = \{a, \{a\}, [a, b, c], d\} \quad Z = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}$$

1. (21 points) Write out each of the sets listed below.

- a) $X \cup Y$
- b) $X \cup \emptyset$
- c) $Z \cup \emptyset$
- d) $X \cap \emptyset$
- e) $X - Y$
- f) $P(X)$
- g) $X \times Y$

2. (18 points) State whether the following propositions are TRUE or FALSE.

- | | | |
|----------------------|----------------------------|--------------------------------|
| a) $a \in X$ | b) $\{a\} \in X$ | c) $a \in Y$ |
| d) $\{a\} \in Y$ | e) $\{a\} \subseteq X$ | f) $\{a\} \subseteq Y$ |
| g) $\emptyset \in Z$ | h) $\emptyset \subseteq Z$ | i) $\{[a, a]\} \in X \times X$ |

3. (20 points) Suppose that sets X and Y have n and m elements, respectively. How many elements do the following sets have? Explain your answer. No points will be given to answers without accompanying explanations.

- a) $X \cup Y$ b) $X \cap Y$ c) $X \times Y$ d) $P(X)$

4. (21 points) Consider a set $X = \{-2, -1, 0, 1, 2, 3\}$ and a relation R defined on X such that $aRb \Leftrightarrow |a| = |b|$. Prove that R is an equivalence relation and show the equivalence classes of R .

5. (20 points) Consider an arbitrary set P of people wearing hats and jackets. For simplicity, assume that there are n people in P , and their hats and jackets have a color that is identifiable by a single color name such as red, brown, storm blue, and so on. State whether the following relations are equivalence relations and prove your answer.

a) Relation R_1 where $x, y \in P$ and xR_1y when x and y have the same color hat.

b) Relation R_2 where $x, y \in P$ and xR_2y when x and y have the same color hat or the same color jacket.