CS3311 Lecture Notes: Chapter 19 LL(k) Grammars

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Definition 19.1.1 Let $G = (V, \Sigma, P, S)$ be a CFG and $A \in V$.

i. The **lookahead set of the variable A, LA(A),** is defined by

$$LA(A) = \{x \mid S \overset{*}{\Rightarrow} uAv \overset{*}{\Rightarrow} ux \in \Sigma^*\}$$

ii. For each rule $A \rightarrow w$ in P, the **lookahead set of the rule $A \rightarrow w$** is defined by

$$LA(A \rightarrow w) = \{x \mid wv \overset{*}{\Rightarrow} x \text{ where } x \in \Sigma^* \text{ and } S \overset{*}{\Rightarrow} uAv\}$$

$LA(A)$: all terminal strings derivable from strings Av, where uAv is a left sentential form of the grammar.

$LA(A \rightarrow w)$: is the subset of $LA(A)$ in which the subderivations $Av \overset{*}{\Rightarrow} x$ are initiated with the rule $A \rightarrow w$.

Definition 19.1.2 Let $G = (V, \Sigma, P, S)$ be a CFG and let k be a natural number greater than zero.

i. **trunc$_k$** is a function from $P(\Sigma^*)$ to $P(\Sigma^*)$:

$$\text{trunc}_k(X) = \{u \mid u \in X \text{ with length}(u) \leq k \text{ or } uv \in X \text{ with length}(u) = k\}$$

ii. The **length-k lookahead set of the variable A**:

$$LA_k(A) = \text{trunc}_k(LA(A))$$

iii. The **length-k lookahead set of the rule $A \rightarrow w$**:

$$LA_k(A \rightarrow w) = \text{trunc}_k(LA(A \rightarrow w))$$

The lookahead sets are computed from FIRST and FOLLOW sets.

Definition 19.2.1 Let $G = (V, \Sigma, P, S)$ be a CFG. For every string $u \in (V \cup \Sigma)^*$ and $k > 0$, the set FIRST$_k(u)$ is defined by:

$$\text{FIRST}_k(u) = \text{trunc}_k(\{x \mid u \overset{*}{\Rightarrow} x, x \in \Sigma^*\}).$$

Algorithm 19.4.1
Construction of FIRST$_k$ sets

input: context-free grammar $G = (V, \Sigma, P, S)$
private: F' : the FIRST set from the previous iteration
F : the FIRST set

1. **for** each $a \in \Sigma$ **do** F'(a) := $\{a\}$

2. **for** each $A \in V$ **do**
   if $A \rightarrow \lambda$ is a rule in P **then**
   $F(A) := \{\lambda\}$
   else
   $F(A) := \emptyset$

3. **repeat**
   3.1 **for** each $A \in V$ **do** $F'(A) := F(A)$
   3.2 **for** each rule $A \rightarrow u_1 u_2 \ldots u_n$ with $n > 0$ **do**
   $F(A) := F(A) \cup trunc_k(F'(u_1)F'(u_2)\ldots F'(u_n))$
   until $F(A) = F'(A)$ for all $A \in V$

4. FIRST$_k(A) := F(A)$

return the FIRST sets

Example 19.4.1

$S \to A\#\#$
$A \to aAd \,|\, BC$
$B \to bBc \,|\, \lambda$
$C \to acC \,|\, ad$

Construct the FIRST_1 sets. The first step is the initialization:

$F'(a) = \{a\} \qquad F'(b) = \{b\} \qquad F'(c) = \{c\} \qquad F'(d) = \{d\} \qquad F'(\#) = \{\#\}$

$F(S) = \emptyset \qquad F(A) = \emptyset \qquad F(B) = \{\lambda\} \qquad F(C) = \emptyset$

iteration 1:

rule $S \to A\#\#$

$F(S) = F(S) \cup \text{trunc}_1(F'(A)F'(\#)F'(\#)) = \emptyset \cup \text{trunc}_1(\emptyset\{\#\}\{\#\}) = \emptyset$

rule $A \to aAd$

$F(A) = F(A) \cup \text{trunc}_1(F'(a)F'(A)F'(d)) = \emptyset \cup \text{trunc}_1(\{a\}\emptyset\{d\}) = \emptyset$

rule $A \to BC$

$F(A) = F(A) \cup \text{trunc}_1(F'(B)F'(C)) = \emptyset \cup \text{trunc}_1(\{\lambda\}\emptyset) = \emptyset$

rule $B \to bBc$

$F(B) = F(B) \cup \text{trunc}_1(F'(b)F'(B)F'(c)) = \{\lambda\} \cup \text{trunc}_1(\{b\}\{\lambda\}\{c\}) = \{\lambda, b\}$

rule $C \to acC$

$F(C) = F(C) \cup \text{trunc}_1(F'(a)F'(c)F'(C)) = \emptyset \cup \text{trunc}_1(\{a\}\{c\}\emptyset) = \emptyset$

rule $C \to ad$

$F(C) = F(C) \cup \text{trunc}_1(F'(a)F'(d)) = \emptyset \cup \text{trunc}_1(\{a\}\{d\}) = \{a\}$

iteration 2:

rule $S \to A\#\#$

$F(S) = F(S) \cup \text{trunc}_1(F'(A)F'(\#)F'(\#)) = \emptyset \cup \text{trunc}_1(\emptyset\{\#\}\{\#\}) = \emptyset$

rule $A \to aAd$

$F(A) = F(A) \cup \text{trunc}_1(F'(a)F'(A)F'(d)) = \emptyset \cup \text{trunc}_1(\{a\}\emptyset\{d\}) = \emptyset$

rule $A \to BC$

$F(A) = F(A) \cup \text{trunc}_1(F'(B)F'(C)) = \emptyset \cup \text{trunc}_1(\{\lambda, b\}\{a\}) = \{a, b\}$

rule $B \to bBc$

$F(B) = F(B) \cup \text{trunc}_1(F'(b)F'(B)F'(c)) = \{\lambda, b\} \cup \text{trunc}_1(\{b\}\{\lambda, b\}\{c\}) = \{\lambda, b\}$ (still)

rule $C \to acC$

$F(C) = F(C) \cup \text{trunc}_1(F'(a)F'(c)F'(C)) = \{a\} \cup \text{trunc}_1(\{a\}\{c\}\{a\}) = \{a\}$ (still)

rule $C \to ad$

$F(C) = F(C) \cup \text{trunc}_1(F'(a)F'(d)) = \{a\} \cup \text{trunc}_1(\{a\}\{d\}) = \{a\}$ (still)

iteration 3:

rule $S \rightarrow A\#\#$

$F(S) = F(S) \cup$ trunc$_1(F'(A)F'(\#)F'(\#)) = \emptyset \cup$ trunc$_1(\{a,b\}\{\#\}\{\#\}) = \{a,b\}$

rule $A \rightarrow aAd$

$F(A) = F(A) \cup$ trunc$_1(F'(a)F'(A)F'(d)) = \{a,b\} \cup$ trunc$_1(\{a\}\{a,b\}\{d\}) = \{a,b\}$

rule $A \rightarrow BC$

$F(A) = F(A) \cup$ trunc$_1(F'(B)F'(C)) = \{a,b\} \cup$ trunc$_1(\{\lambda,b\}\{a\}) = \{a,b\}$

rule $B \rightarrow bBc$

$F(B) = F(B) \cup$ trunc$_1(F'(b)F'(B)F'(c)) = \{\lambda,b\} \cup$ trunc$_1(\{b\}\{\lambda,b\}\{c\}) = \{\lambda,b\}$ (still)

rule $C \rightarrow acC$

$F(C) = F(C) \cup$ trunc$_1(F'(a)F'(c)F'(C)) = \{a\} \cup$ trunc$_1(\{a\}\{c\}\{a\}) = \{a\}$ (still)

rule $C \rightarrow ad$

$F(C) = F(C) \cup$ trunc$_1(F'(a)F'(d)) = \{a\} \cup$ trunc$_1(\{a\}\{d\}) = \{a\}$ (still)

FIRST$_1(S) = \{a,b\}$
FIRST$_1(A) = \{a,b\}$
FIRST$_1(B) = \{\lambda,b\}$
FIRST$_1(C) = \{a\}$

We will not be able to distinguish between the C rules with one step lookahead because they both start with a.

Construct the FIRST$_2$ sets. The first step is the initialization:

$F'(a) = \{a\}$ $F'(b) = \{b\}$ $F'(c) = \{c\}$ $F'(d) = \{d\}$ $F'(\#) = \{\#\}$

$F(S) = \emptyset$ $F(A) = \emptyset$ $F(B) = \{\lambda\}$ $F(C) = \emptyset$

iteration 1:

rule $S \rightarrow A\#\#$

$F(S) = F(S) \cup$ trunc$_2(F'(A)F'(\#)F'(\#)) = \emptyset \cup$ trunc$_2(\emptyset\{\#\}\{\#\}) = \emptyset$

rule $A \rightarrow aAd$

$F(A) = F(A) \cup$ trunc$_2(F'(a)F'(A)F'(d)) = \emptyset \cup$ trunc$_2(\{a\}\emptyset\{d\}) = \emptyset$

rule $A \rightarrow BC$

$F(A) = F(A) \cup$ trunc$_2(F'(B)F'(C)) = \emptyset \cup$ trunc$_2(\{\lambda\}\emptyset) = \emptyset$

rule $B \rightarrow bBc$

$F(B) = F(B) \cup$ trunc$_2(F'(b)F'(B)F'(c)) = \{\lambda\} \cup$ trunc$_2(\{b\lambda c\}) = \{\lambda, bc\}$

rule $C \rightarrow acC$

$F(C) = F(C) \cup$ trunc$_2(F'(a)F'(c)F'(C)) = \emptyset \cup$ trunc$_2(\{a\}\{c\}\emptyset) = \emptyset$

rule $C \rightarrow ad$

$F(C) = F(C) \cup$ trunc$_2(F'(a)F'(d)) = \emptyset \cup$ trunc$_2(\{a\}\{d\}) = \{ad\}$

iteration 2:

rule $S \to A\#\#$

$F(S) = F(S) \cup \text{trunc}_2(F'(A)F'(\#)F'(\#)) = \emptyset \cup \text{trunc}_2(\emptyset\{\#\}\{\#\}) = \emptyset$

rule $A \to aAd$

$F(A) = F(A) \cup \text{trunc}_2(F'(a)F'(A)F'(d)) = \emptyset \cup \text{trunc}_2(\{a\}\emptyset\{d\}) = \emptyset$

rule $A \to BC$

$F(A) = F(A) \cup \text{trunc}_2(F'(B)F'(C)) = \emptyset \cup \text{trunc}_2(\{\lambda, bc\}\{ad\}) = \{ad, bc\}$

rule $B \to bBc$

$F(B) = F(B) \cup \text{trunc}_2(F'(b)F'(B)F'(c)) = \{\lambda, bc\} \cup \text{trunc}_2(\{b\}\{\lambda, bc\}\{c\}) = \{\lambda, bc, bb\}$

rule $C \to acC$

$F(C) = F(C) \cup \text{trunc}_2(F'(a)F'(c)F'(C)) = \{ad\} \cup \text{trunc}_2(\{a\}\{c\}\{ad\}) = \{ad, ac\}$

rule $C \to ad$

$F(C) = F(C) \cup \text{trunc}_2(F'(a)F'(d)) = \{ad, ac\} \cup \text{trunc}_2(\{a\}\{d\}) = \{ad\}$ (RHS is only terminals)

iteration 3:

rule $S \to A\#\#$

$F(S) = F(S) \cup \text{trunc}_2(F'(A)F'(\#)F'(\#)) = \emptyset \cup \text{trunc}_2(\{ad, bc\}\{\#\}\{\#\}) = \{ad, bc\}$

rule $A \to aAd$

$F(A) = F(A) \cup \text{trunc}_2(F'(a)F'(A)F'(d)) = \{ad, bc\} \cup \text{trunc}_2(\{a\}\{ad, bc\}\{d\}) = \{ad, bc, aa, ab\}$

rule $A \to BC$

$F(A) = F(A) \cup \text{trunc}_2(F'(B)F'(C)) = \{ad, bc, aa, ab\} \cup \text{trunc}_2(\{\lambda, b\}\{a\}) = \{ad, bc, aa, ab, bb, ac\}$

rule $B \to bBc$

$F(B) = F(B) \cup \text{trunc}_2(F'(b)F'(B)F'(c)) = \{\lambda, bc, bb\} \cup \text{trunc}_2(\{b\}\{\lambda, bc, bb\}\{c\}) = \{\lambda, bc, bb\}$ (still)

rule $C \to acC$

$F(C) = F(C) \cup \text{trunc}_2(F'(a)F'(c)F'(C)) = \{ad, ac\} \cup \text{trunc}_2(\{a\}\{c\}\{ad, ac\}) = \{ad, ac\}$ (still)

rule $C \to ad$

$F(C) = F(C) \cup \text{trunc}_2(F'(a)F'(d)) = \{ad, ac\} \cup \text{trunc}_2(\{a\}\{d\}) = \{ad, ac\}$ (still)

iteration 4:

rule $S \to A\#\#$

$F(S) = F(S) \cup$ trunc$_2(F'(A)F'(\#)F'(\#)) = \emptyset \cup$ trunc$_2(\{ad, bc, aa, ab, bb, ac\}\{\#\}\{\#\}) = \{ad, bc, aa, ab, bb, ac\}$

others don't change:

rule $A \to aAd$

$F(A) = F(A) \cup$ trunc$_2(F'(a)F'(A)F'(d)) =$
$\{ad, bc, aa, ab, bb, ac\} \cup$ trunc$_2(\{a\}\{ad, bc, aa, ab, bb, ac\}\{d\}) = \{ad, bc, aa, ab, bb, ac\}$

rule $A \to BC$

$F(A) = F(A) \cup$ trunc$_2(F'(B)F'(C)) =$
$\{ad, bc, aa, ab, bb, ac\} \cup$ trunc$_2(\{\lambda, b\}\{a\}) = \{ad, bc, aa, ab, bb, ac\}$

rule $B \to bBc$

$F(B) = F(B) \cup$ trunc$_2(F'(b)F'(B)F'(c)) = \{\lambda, bc, bb\} \cup$ trunc$_2(\{b\}\{\lambda, bc, bb\}\{c\}) = \{\lambda, bc, bb\}$

rule $C \to acC$

$F(C) = F(C) \cup$ trunc$_2(F'(a)F'(c)F'(C)) = \{ad, ac\} \cup$ trunc$_2(\{a\}\{c\}\{ad, ac\}) = \{ad, ac\}$

rule $C \to ad$

$F(C) = F(C) \cup$ trunc$_2(F'(a)F'(d)) = \{ad, ac\} \cup$ trunc$_2(\{a\}\{d\}) = \{ad, ac\}$

FIRST$_2(S) = \{ad, bc, aa, ab, bb, ac\}$
FIRST$_2(A) = \{ad, bc, aa, ab, bb, ac\}$
FIRST$_2(B) = \{\lambda, bc, bb\}$
FIRST$_2(C) = \{ad, ac\}$