Computing with Geometry as an Undergraduate Course

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Why Is It Important?

- **This is a *geometric* world**

- ***Geometry* plays a central role in many areas in computer science and engineering:**

  computer graphics, computer−aided design, geometric modeling, computer vision, manufacturing, robotics, GIS, ...

- **The skill of handling geometric objects is virtually missing in a typical computer science curriculum**

- **The Computer Science and Tele−communication Board and National Research Council suggested that more *continuous* math should be taught in CS**

Design Merit

- The underlying *theme* is

  Geometry → Representation
  → Algebra
  → Algorithm
  → Program

- Intuitive, less mathematical and elementary

- Hands−on and learning−by−doing

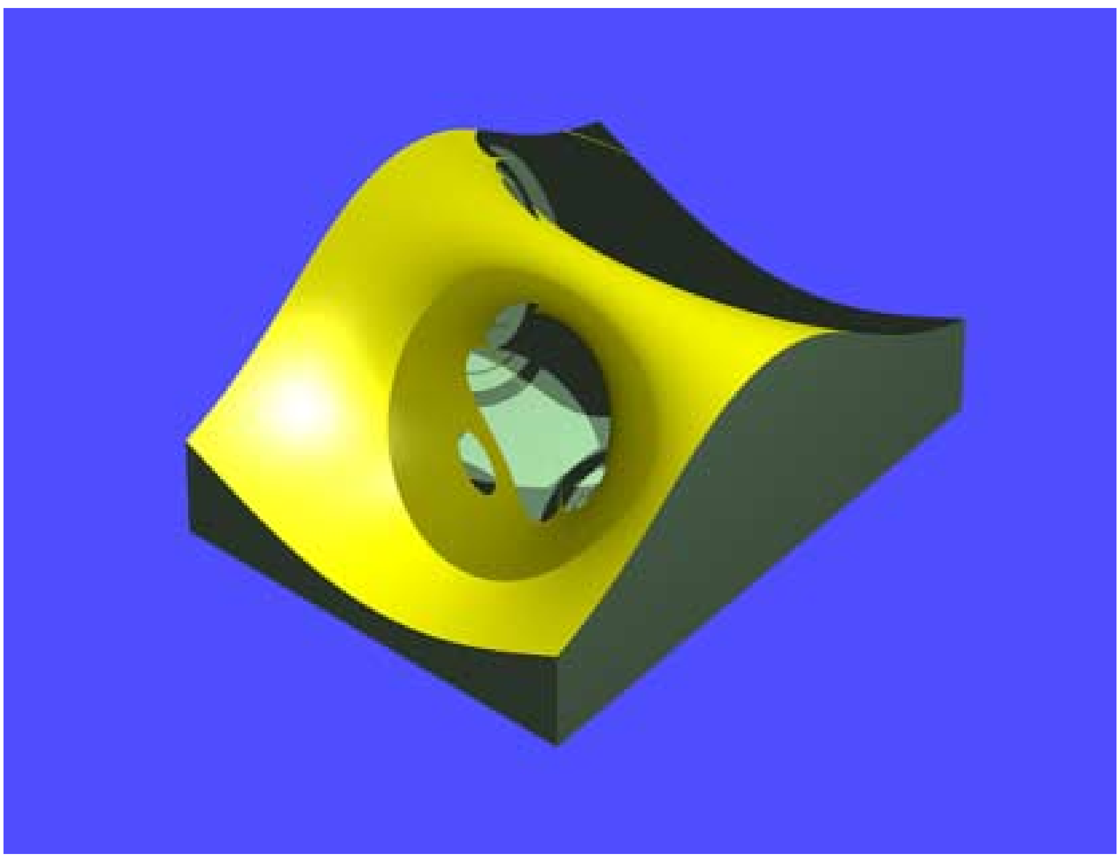
- Using DesignMentor: A tool for teaching curve and surface design

- Prerequisites: calculus and linear algebra

Unit 1: Course Overview

- The *theme* of this course
- The complexity of a geometric problem

  Dimensional, analytic, combinatorial
- The impact of float point calculation on geometric problems



A poor equation solver can achieve this easily!!

Unit 2: Basic Geometric Concept

- **Euclidean Geometry**

  Coordinate, Euclidean transformations and their matrices, quaternions

- **Affine Geometry**

  Affine transformations (*e.g.*, scaling, shear) and their matrices, affine space

- **Projective Geometry**

  The concept of points and lines at infinity, projective transformations and their matrices, projective space, cross−ratio

- **Floating Point Computation**

  Lossing of significant digits, error cumulation, problems with commutative law and distributive law

- **Examples and Applications**

Unit 3: Object Representations

Wireframes:

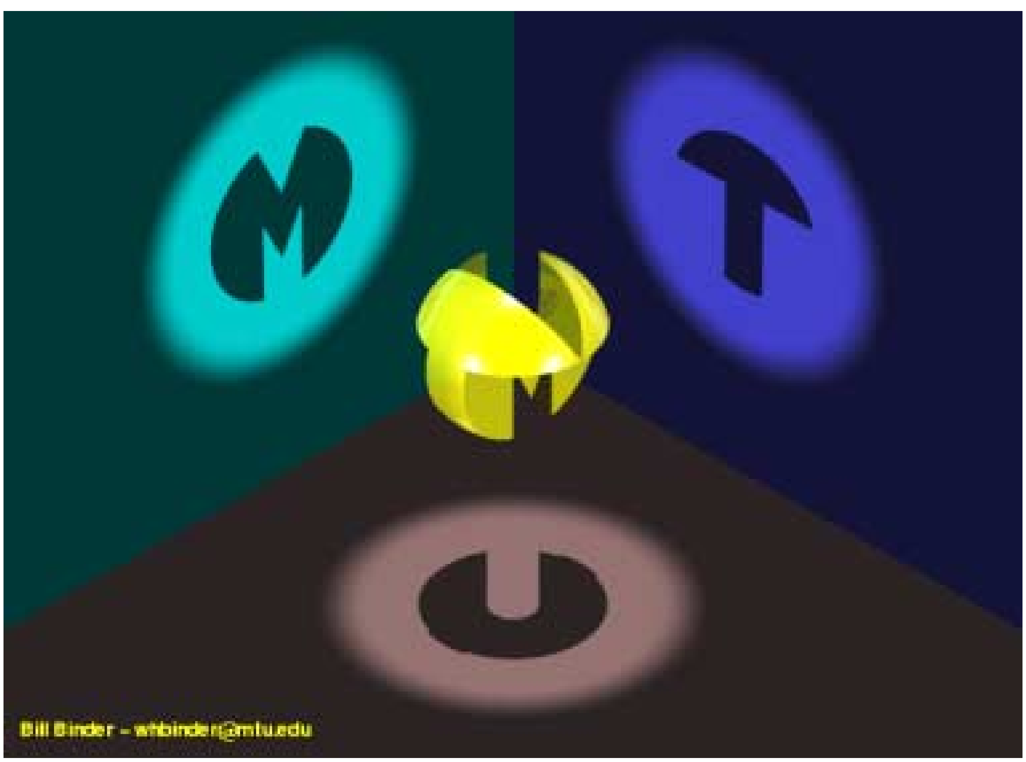
Advantages, disadvantages, ambiguity

Boundary Representations:

Manifolds, winged−edge representations,
Euler−Poincare characteristic, Euler operators,
non−manifolds

Constructive Solid Geometry:

Interior, exterior and closure, regularized
Boolean operators, CSG solid design



Student work on CSG Design

Unit 4: Parametric Curves and Surfaces

- **Polynomial and rational curves**
- **The moving triad**

  tangent, bi−normal, normal vectors
- **Curvature and curvature sphere**
- **Singular and Inflection Points**
- **Tangential and Geometric Continuity**
- **The Meaning of Uniformization**

  Not all curves are polynomial
- **Discussions**

  Classification of conics using the line at infinity
  The proof of circle being non−polynomial

Unit 5: Bezier, B−spline and NURBS Curves

- **Motivation and Important Properties:**

  Control points, partition of unity, convex hull, variation diminishing and affine invariance

- **Bezier Curves:**

  Construction, editing, de Casteljau’s algorithm, derivatives, subdivision and degree elevation

- **B−spline Curves**

  B−spline basis, construction, local modification, strong convex hull, derivatives

- **NURBS** ***N**on−**U**niform **R**ational **B**−**S**pline*

  Motivation, meaning of weights, NURBS basis, a NURBS curve as the projection of a 4D B−spline curve to 3D, infinite control points

Unit 6: Advanced Geometric Algorithms

- **Knots Revisited**
  - **The meaning of knots**
- **Knot Insertion**
  - **Inserting a new knot *without* changing the shape of the curve, single insertion, multiple insertion**
- **De Boor's Algorithm**
  - **De Boor's algorithm via knot insertion, and de Casteljau's algorithm as a special case**
- **Curve Subdivision**
- **Degree Elevation**

Unit 7: Parametric, Bezier, B−Spline, and NURBS Surfaces

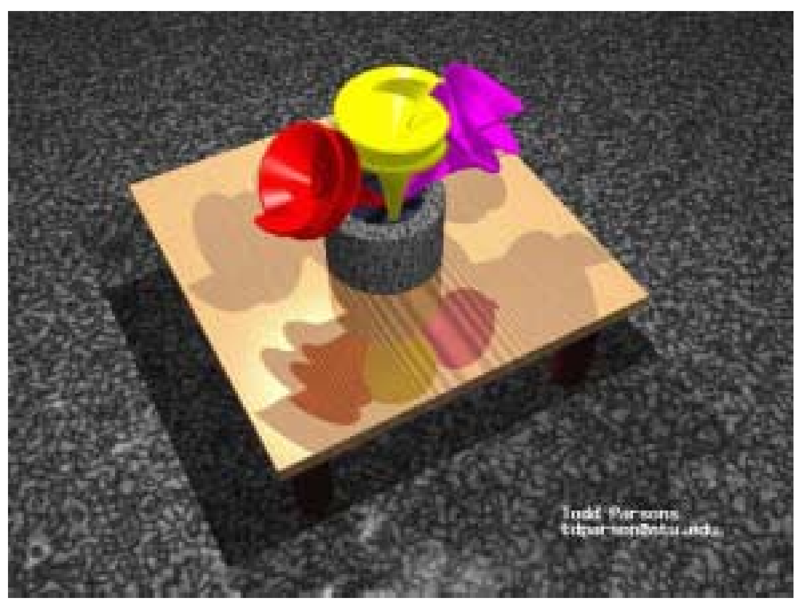
Basic Concepts

Surface normal, tangent plane, naive surface triangulation, isoparametric curves, tensor product surfaces

Bezier, B−Spline and NURBS Surfaces

Surface construction from two curves, 2D basis functions, 3D important properties from those of 2D, de Casteljau’s and de Boor’s algorithms for surfaces

A Twisted Sphere



Dini’s Surface

Naive Surface Triangulation: Student Work

Unit 8: Cross−Sectional Design

What is Cross−Sectional Design

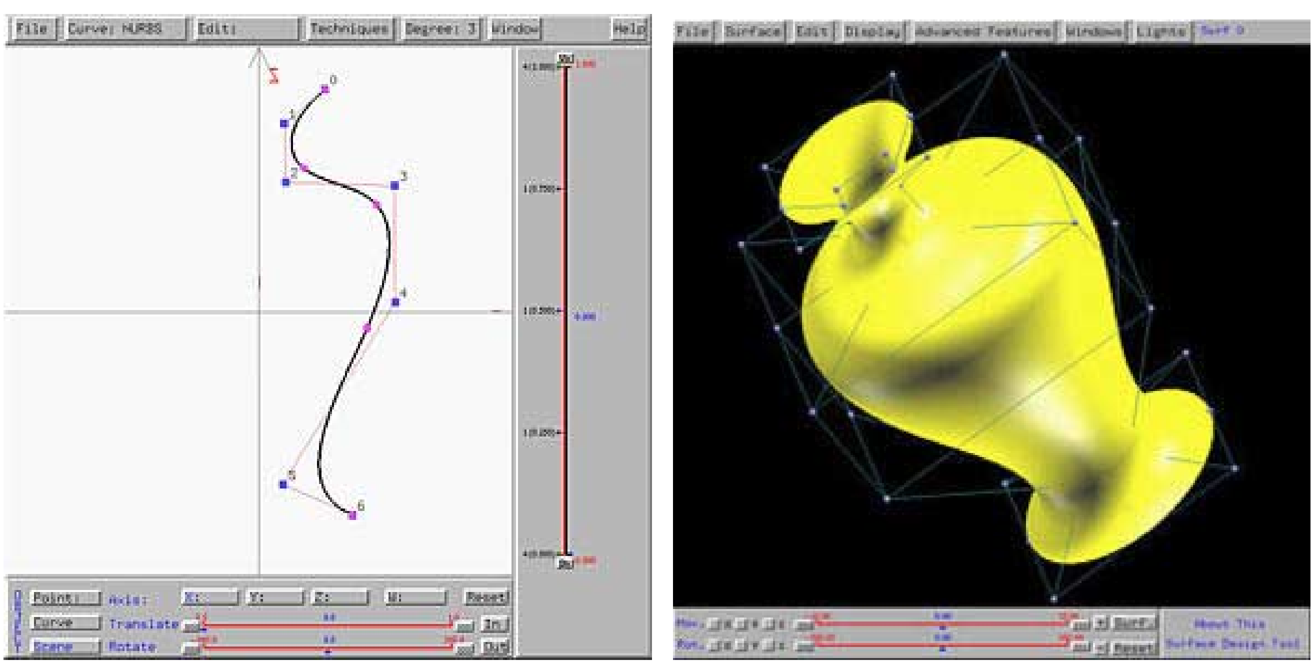
Creating surfaces using curves, profile and trajectory curves, compatible curves

Cross−Sectional Design Surfaces

Ruled surfaces, surfaces of revolution, swung surfaces, simple swept surfaces, and skinned surfaces

Interpolation Surfaces

Swept surfaces via skinning, interpolating a curve network (*i.e.*, Gordan surfaces)



Generating a surface of revolution

Unit 9: Algorithm Robustness

- **Loss of Significant Digits**

  Imprecise input, cumulation of errors in geometric transformations and computation

- **Various Computation Schemes**

  Exact (Symbolic), Approximation, and Interval arithmetic

- **Robust Algorithm Design**

- **Well−Known Experiments**

  Equation solvers, Sturm sequences, Dobkin’s growing/shrinking pentagons, Euclidean transformations

Course Evaluation

- **This elective course has been taught three times to junior/senior students**

- **The following is a student self assessment survey**

	Mean	*Var*
Pre−Test	**17.80**	**7.83**
Post−Test	**49.37**	**6.07**
Gain	**32.56**	**8.81**

  - **17 questions were asked *before* and *after* the course**
  - **Self assessment levels range from 0 (no understanding) to 4 (excellent understanding)**
  - **The average gain is statistically significant**

- **Students like the non−mathematical approach and our software tools**

Results and Dissemination

Available Materials

A course electronic book, a software tool **DesignMentor**, a set of user guides and publications

`http://www.cs.mtu.edu/~shene/NSF-2`

Dissemination Statistics

- (visitors)

	Daily Avg
Course Info Page	9.8
Electronic Book	13.0
Curve User Guide	5.3
Surface User Guide	3.1

- There are 900+ downloads of **DesignMentor**

CS	*Sci*	*Eng*	EDU *Other*	COM	*Other*
28.7	3.0	7.7	11.1	26.9	21.6

N. Amer	*S. Amer*	*Europe*	*Far East*	*Other*
41.5	2.7	38.5	8.6	8.7

Future Work

- **Interpolation and Approximation**

  Regular and scattered data

- **Triangular Patches**

  Bezier triangles, triangular B−splines, and multi−sided patches

- **Curve and Surface Interrogation**

- **Implicit Curves and Surfaces**

- **The Blossoming Principle**

- **Important Geometric Operations**

  Surfaces blending, curve and surface intersection, curve tracing, ...